Math 241, Final exam, Summer 2002

PRINT Your Name:______________________________

There are 20 problems on 10 pages. Each problem is worth 5 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

I will not grade this exam until Friday. **Get your course grade from VIP.** The grade will be available from VIP as soon as I finish grading the exams.

I will post an answer key on my web site: www.math.sc.edu; click on faculty directory; click on kustin; click on teaching; click on math 241. The key will be posted shortly after the exam is completed.

1. Let $f(x,y) = x\sin(xy)$. Find $\vec{\nabla} f$.

2. Find the equations of the line through the points $P = (1,-3,4)$ and $Q = (3,4,6)$. **Check your answer.**

3. Find the equation of the plane through the points $P = (2,1,2)$, $Q = (3,3,6)$, and $R = (0,-1,0)$. **Check your answer.**

4. Let $f(x,y) = \dfrac{x^2}{x^2+2y^2}$. Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along $y = 3x$.

5. Identify all local extreme points and all saddle points of $f(x,y) = x^2y - 6y^2 - 3x^2$.

6. Find the intersection of the two lines:
$$\frac{x-5}{2} = \frac{y-3}{1} = \frac{z}{-1} \quad \text{and} \quad \frac{x+8}{3} = \frac{y+5}{2} = \frac{z+1}{1}.$$
Check your answer.

7. The temperature of a plate at the point (x,y) is $T(x,y) = 20 - 2x^2 - y^2$.
   (a) Draw and label the level sets $T = -7$, $T = 0$, $T = 10$, and $T = 20$
   (b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(3,3)$. Draw the path of the particle.
   (c) Find the equation which gives the path of the particle of part (b).

8. The position of a moving particle at time t is given by the position vector
$$\vec{r}(t) = 3\sin t\,\vec{i} + 4\cos t\,\vec{j}\,.$$
   (a) Graph the path of the object.
   (b) Eliminate the parameter and express the path of the object in cartesian coordinates.
   (c) Which point on the curve corresponds to $t = \frac{\pi}{4}$?
   (d) Draw $\vec{v}(\frac{\pi}{4})$. Put the tail on your answer to (c).
   (e) Draw $\vec{a}(\frac{\pi}{4})$. Put the tail on your answer to (c).

9. Compute the directional derivative $D_{\vec{u}} f$ at the point $(3,2)$ in the direction of the unit vector $\vec{\boldsymbol{u}} = \frac{5}{13}\vec{\boldsymbol{i}} + \frac{12}{13}\vec{\boldsymbol{j}}$ for $f(x,y) = 3x^2y^4$.

10. Where does the line normal to $x^2 + 2y^2 + 3z^2 = 9$ at $(2,1,-1)$ intersect $2x + y - z + 3 = 0$?

11. Compute $\iint\limits_R (x^2+2y)\,dA$, where R is the region between $y = x^2$ and $y = \sqrt{x}$.

12. Find the volume of the solid which is between $z = 16 - x^2 - y^2$ and the xy-plane.

13. Compute $\iint\limits_R x^2\,dA$, where R is the region between $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

14. Compute $\int_0^2 \int_0^{\sqrt{4-y^2}} x^2\,dx\,dy$.

15. Compute $\int_C (x+y+z)\,dx + x\,dy - yz\,dz$, where C is the line segment from $(1,2,1)$ to $(2,1,0)$.

16. Let $\vec{\boldsymbol{a}} = 1\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 3\vec{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = 4\vec{\boldsymbol{i}} + 4\vec{\boldsymbol{j}} + 10\vec{\boldsymbol{k}}$. Find vectors $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$ with $\vec{\boldsymbol{b}} = \vec{\boldsymbol{u}} + \vec{\boldsymbol{v}}$, $\vec{\boldsymbol{u}}$ parallel to $\vec{\boldsymbol{a}}$, and $\vec{\boldsymbol{v}}$ perpendicular to $\vec{\boldsymbol{a}}$. (Every number in the answer is an integer. If you have fractions, either you can rid of them or you have made a mistake.) **Check your answer.**

17. Graph and name $x^2 + y^2 - z^2 = 1$ in 3-space.

18. Graph and describe the graph of $yz = 0$ in 3-space.

19. Find the equation of the line tangent to the curve parameterized by $\vec{\boldsymbol{r}}(t) = 3t^2\vec{\boldsymbol{i}} + t^3\vec{\boldsymbol{j}}$ at $t = 2$.

20. Find the equation of the plane tangent to $z = x^2 + y^2$ at the point where $x = 3$ and $y = 4$.