Math 241, Spring 1998, final

PRINT Your Name:______________________________

There are 15 problems on 6 pages. Each problem is worth 10 points. SHOW your work. *CIRCLE* your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Find the equations of the line which contains the points $(4, 2, -3)$ and $(4, -3, 0)$.

2. Find the equation of the plane which contains the points $(4, 2, -3)$, $(4, -3, 0)$, and $(2, 3, 4)$.

3. Let $\vec{\boldsymbol{a}} = 4\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = 2\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}} + 3\vec{\boldsymbol{k}}$. Find vectors $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$ with $\vec{\boldsymbol{b}} = \vec{\boldsymbol{u}} + \vec{\boldsymbol{v}}$, $\vec{\boldsymbol{u}}$ parallel to $\vec{\boldsymbol{a}}$, and $\vec{\boldsymbol{v}}$ perpendicular to $\vec{\boldsymbol{a}}$.

4. Find the area inside $r = 4\sin\theta$.

5. Find the volume of the solid which is bounded by $z = x^2 + y^2$ and $z = \sqrt{1 - x^2 - y^2}$.

6. Describe and graph $\vec{\boldsymbol{r}}(t) = t\,\vec{\boldsymbol{i}} + \sin t\,\vec{\boldsymbol{j}} + \cos t\,\vec{\boldsymbol{k}}$.

7. Describe and graph $z = x^2 - y^2$.

8. Find the equations of the line tangent to $\vec{\boldsymbol{r}}(t) = e^2 t\,\vec{\boldsymbol{i}} + t\,\vec{\boldsymbol{j}} - \sqrt{4t^2 - 1}\,\vec{\boldsymbol{k}}$ at $t = 1$.

9. Find the equation of the plane tangent to $z = 2x^2 + y^2$ at the point where $x = 3$ and $y = 2$.

10. Compute $\int_C (12xy)\,dx + (6x^2 - 7e^y + 2y)\,dy$ where C consists of three line segments. The first line segment for C starts at (1,2) and goes to (8,75); the second segment is from (8,75) to (198,706); and the third segment is from (198,706) to (3,4).

11. Find the volume of the solid bounded by $z = 18 - x^2 - y^2$ and $z = x^2 + y^2 - 18$.

12. Identify all local extreme points and all saddle points of $f(x, y) = x^2 y - 6y^2 - 3x^2$.

13. Find the global extreme points of $f(x, y) = 3x + 4y$, which is defined on $S = \{(x, y) \mid 0 \le x \le 1, -1 \le y \le 1\}$.

14. Where does $\dfrac{x-1}{2} = \dfrac{y-2}{-1}$, $z = 3$ intersect $4x + 3y + 2z = 6$?

15. If the temperature of a plate at the point (x, y) is $T(x, y) = 10 + 2x^2 - y^2$, then find the path a heat-seeking particle (which always moves in the direction of greatest increase of temperature) would follow if it starts at $(4, 2)$.