**Math 241, Spring 2001, Exam 4**

PRINT Your Name:______________________________

There are 10 problems on 6 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1.
   (a) Find $\lim\limits_{\substack{(x,y)\to(0,0)\\ \text{along } y=3x}} \frac{x^2y}{x^4+y^2}$ .

   (b) Find $\lim\limits_{\substack{(x,y)\to(0,0)\\ \text{along } y=2x^2}} \frac{x^2y}{x^4+y^2}$ .

   (c) What is $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ ? Why?

2. Let $R$ be the region $R = \{(x,y) \mid 2 \le x \le 8, \text{ and } 2 \le y \le 6\}$ . Let $P$ be the partition of $R$ into six equal squares by the lines $x = 4$ , $x = 6$ , and $y = 4$ . Approximate $\iint\limits_R (72 - x^2 - y)\,dA$ by calculating the corresponding Riemann sum $\sum\limits_{k=1}^{6} f(\bar{x}_k, \bar{y}_k)\Delta A_k$ , where $(\bar{x}_k, \bar{y}_k)$ is the center of the $k^{\text{th}}$ box, and $\Delta A_k$ is the area of the $k^{\text{th}}$ box. (Be sure to answer the question I have asked. You will receive no credit for computing the integral directly. Express your answer as a sum of products. There is no need to do any arithmetic.)

3. Identify all local maximum points, all local maximum points, and all saddle points of $f(x,y) = 2x^4 - x^2 + 3y^2$ .

4. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 60 inches and is increasing at 4 inches per minute and the radius is 30 inches and is increasing at 3 inches per minute. How fast is the volume increasing at that instant? (The volume of a cone is $V = (1/3)\pi r^2 h$ .)

5. Find $\int_0^{\pi/2} \int_0^1 x \sin xy \, dy\, dx$ .

6. Find $\int_{1/2}^{1} \int_0^{2x} \cos(\pi x^2)\, dy\, dx$ .

7. Evaluate $\iint_R \sin(y^3) dA$ , where $R$ is the region bounded by $y = \sqrt{x}$ , $y = 2$ , and $x = 0$ .

8. Consider the solid which is bounded by $x + 3y + 6z = 12$ and the three coordinate planes. Find the volume of the solid. Set up the integral, **but do NOT compute the integral.**

9. Evaluate $\iint_R e^{x^2+y^2} dA$ , where $R$ is the region enclosed by $x^2 + y^2 = 4$ .