9. (14 points) The temperature of a plate at the point \((x, y)\) is \(T(x, y) = x^2 + 4y^2\).

(a) Draw and label the level sets \(T = 0, T = 4, T = 36, T = 64\).

(b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point \((\sqrt{3}, \frac{1}{2})\). Draw the path of the particle.

(c) Find the equation which gives the path of the particle of part (b).

\[
\begin{align*}
\text{Let } & F(x, y) = x^2 + 4y^2, \text{ then } \frac{\partial F}{\partial x} = 2x, \frac{\partial F}{\partial y} = 8y. \\
\text{We know that } & F'(c) = \frac{\partial F}{\partial x} \cdot T_x + \frac{\partial F}{\partial y} \cdot T_y \\
& x'(c) + 4y'(c)^2 = 2(x'(c) + 8y'(c)^2) \\
& \frac{x'(c)}{2y(c)} = \frac{y(c)}{8y(c)} \\
& \therefore \frac{4y(c)}{x'(c)} = \frac{y'(c)}{8y(c)} \\
& \text{Apply } S \text{-dt to both sides} \\
& 4 \ln |x'(c)| + R = \ln |y'(c)| \\
& \text{Apply e}^{-} \text{ to both sides} \\
& e^{-R} \left( e^{\ln |x'(c)|} \right)^4 = e^{\ln |y'(c)|} \\
& e^{R} (x(c))^4 = y(c) \\
& \text{so } K x^4 = y \quad \text{when } K = \pm e^R \\
& \text{But } (\sqrt{3}, \frac{1}{2}) \text{ is on the curve, so} \\
& K 9 = \frac{1}{2} \\
& K = \frac{1}{18} \\
& y = \frac{1}{18} x^4.
\end{align*}
\]