Math 241, Final Exam, Spring, 2022

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 100 points; each problem is worth 10 points.

Make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

No Calculators, Cell phones, computers, notes, etc.

(1) Find the equation of the plane which contains the lines

$$\begin{cases} x = 1 + 2t \\ y = 2 + 3t \\ z = 3 + 4t \end{cases} \quad \text{and} \quad \begin{cases} x = -1 + 2t \\ y = 1 + 3t \\ z = 2 + 4t, \end{cases}$$

if such a plane exists. Be sure to verify that both lines are on your plane. If no such plane exists, explain why not.

(2) An object moves on the xy-plane. The position vector of the object at time t is $\overrightarrow{\boldsymbol{r}}(t) = 2t^2\,\overrightarrow{\boldsymbol{i}} + 3t^3\,\overrightarrow{\boldsymbol{j}}$. How far does the object travel between $t = 0$ and $t = 1$?

(3) Let f be the function $f(x, y) = x^2 + 3y^2$, P be the point $P = (3, 4)$, and $\overrightarrow{\boldsymbol{u}}$ be the vector $\overrightarrow{\boldsymbol{u}} = 2\,\overrightarrow{\boldsymbol{i}} + 3\,\overrightarrow{\boldsymbol{j}}$.

(a) Draw a few level sets for the function $z = f(x, y)$. In particular, be sure to draw the level set which contains the point P.

(b) Calculate the gradient of f at P.

(c) Draw the gradient of f at P on your answer to (a); put the tail of this gradient on P.

(4) Calculate the directional derivative of the function $f(x, y) = x^2 + 3y^2$ in the direction of $\overrightarrow{\boldsymbol{u}} = 2\,\overrightarrow{\boldsymbol{i}} + 3\,\overrightarrow{\boldsymbol{j}}$ at the point $P = (3, 4)$.

(5) Graph, name, and describe the set of points in three space which satisfy the equation $x^2 + y^2 - z^2 = 1$.

(6) Find the absolute maxima and absolute minima of

$$f(x, y) = x^2 - xy + y^2 + 1$$

on the closed triangular region in the first quadrant bounded by the lines $x = 0$, $y = 4$, and $y = x$.

(7) Find the points on the ellipse $x^2 + 2y^2 = 1$ where $f(x, y) = xy$ has its extreme values.

(8) Find the volume of the solid on and above the xy-plane bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = -y$.

(9) Find the volume of the solid between the surfaces $z = (x^2 + y^2)^2 - 1$ and $z = 4 - 4(x^2 + y^2)$.

(10) An object moves in 3-space. The acceleration vector of the object at time t is $\overrightarrow{\boldsymbol{r}}''(t) = -(3\cos t)\,\overrightarrow{\boldsymbol{i}} - (3\sin t)\,\overrightarrow{\boldsymbol{j}} + 2\,\overrightarrow{\boldsymbol{k}}$; the initial position vector of the object is $\overrightarrow{\boldsymbol{r}}(0) = 4\,\overrightarrow{\boldsymbol{i}}$; and the initial velocity vector of the object is $\overrightarrow{\boldsymbol{r}}'(0) = 3\,\overrightarrow{\boldsymbol{j}}$. Find the position vector of the object at time t.