**Math 241, Final Exam, Spring, 2019**

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

**No Calculators, Cell phones, computers, notes, etc.**

(1) Let $f(x,y) = x\sqrt{x\cos y + 3x^2}$. Find $\frac{\partial f}{dx}$.

(2) Find the maximum and minimum values of $f(x,y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.

(3) Find the equation of the plane tangent to $z = 2x^2 + 3y^2$ at the point where $x = 1$ and $y = 2$.

(4) Find the directional derivative of the function $f(x,y) = 2x^2 + 3y^2$ in the direction of $\overrightarrow{\boldsymbol{v}} = 5\overrightarrow{\boldsymbol{i}} + 3\overrightarrow{\boldsymbol{j}}$ at the point where $x = 1$ and $y = 2$.

(5) Find and identify local extreme points and the saddle points of $f(x,y) = 2x^3 + 9xy^2 + 15x^2 + 27y^2$.

(6) Find the equation of the plane that contains the points $(0,1,2)$, $(1,1,0)$, and $(3,0,1)$.

(7) Find the length of the graph for $y = x^{3/2}$ on the closed interval $1 \le x \le 4$.

(8) Find the work done by the force $\overrightarrow{\boldsymbol{F}}(t) = 2y\overrightarrow{\boldsymbol{i}} + (2x+z)\overrightarrow{\boldsymbol{j}} + (y+2z)\overrightarrow{\boldsymbol{k}}$ as it moves an object along the curve parameterized by the position vector $\overrightarrow{\boldsymbol{r}}(t) = 2\cos t\,\overrightarrow{\boldsymbol{i}} + 2\sin t\,\overrightarrow{\boldsymbol{j}} + t\,\overrightarrow{\boldsymbol{k}}$ from $t = 0$ to $t = 2\pi$.

(9) Find the volume of the region below $x^2 + y^2 + z^2 = 1$ and above $z = \sqrt{x^2 + y^2}$.

(10) Find the area of the region in the $xy$-plane which is bounded by $x + y = 2$ and $x + 4 = y^2$.