Math 241, Final Exam, Fall 2019

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

No Calculators, Cell phones, computers, notes, etc.

(1) An object moves along a curve in the xy-plane. At time zero, the object is at the origin and has velocity $4\vec{\boldsymbol{i}} + \vec{\boldsymbol{j}}$. The acceleration of the object at time t is $4e^{2t}\vec{\boldsymbol{i}} + \frac{3}{\sqrt{t}}\vec{\boldsymbol{j}}$. What is the position vector of the object at time t?

(2) What are the equations of the line tangent to the curve $\vec{\boldsymbol{r}}(t) = t^2\vec{\boldsymbol{i}} + 2t^3\vec{\boldsymbol{j}} + 3t^4\vec{\boldsymbol{k}}$ at $t = 1$?

(3) An object moves along the curve $\vec{\boldsymbol{r}}(t) = t\vec{\boldsymbol{i}} + \frac{2}{3}t^{3/2}\vec{\boldsymbol{j}}$ from $t = 1$ to $t = 2$. How far did the object travel?

(4) Write $3x^2 - 24x + 2y^2 + 20y + 92 = 0$ in the form

$$\frac{(x-h)^2}{a} + \frac{(y-k)^2}{b} = 1$$

for numbers h, k, a, b. (Please make sure that your answer is correct.)

(5) Find the equation of the plane that contains the points $(1, 2, 3)$, $(2, -1, 3)$ and $(3, 2, 0)$. (Please make sure that your answer is correct.)

(6) Find all local extreme points and all saddle points of $f(x, y) = 4 + x^3 + y^3 - 3xy$

(7) Find the absolute maximum and the absolute minimum of the function $f(x, y) = x^2 + xy + y^2 - 3x + 3y$ on the triangular region bounded by the x-axis, the y-axis, and $x + y = 4$.

(8) Find the area of the region bounded by $y = x^2$ and $y = x + 2$.

(9) Compute $\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2+y^2}}\, dx\, dy$.

(10) Find the volume of the solid whose top is $x^2 + y^2 + z^2 = 1$ and whose bottom is the top part of $3z^2 = x^2 + y^2$.