Math 241, Fall 1999, final exam

PRINT Your Name:________________________________

There are 22 problems on 11 pages. Problems 1 through 4 are each worth 6 points. Problems 5 through 22 are each worth 7 points. The exam is worth a total of 150 points. SHOW your work. *CIRCLE* your answer. **NO CALCULATORS!**

1. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equation of the plane through $(1,2,3)$, $(2,0,2)$, and $(-3,1,1)$.

2. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equations of the line through $(5,4,2)$ and $(3,4,7)$.

3. Graph and name $y^2 - x^2 = 1$ in $2-$ space.

4. Graph and name $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} = 1$ in $3-$ space.

5. What are the equations of the line tangent to the curve which is parameterized by $\vec{r}(t) = (3t^2+2)\vec{i} + 6t\vec{j} + (4t^3+2t)\vec{k}$ at $t=1$?

6. Find the equation of the plane tangent to the surface $z = x^2 + 2y^3$ at the point where $x = 2$ and $y = -2$.

7. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\vec{a} = 2\vec{i} + 4\vec{j} + 6\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + \vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$.

8. Find the equations of **any** line which **intersects** and is **perpendicular** to $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-5}{6}$. Be sure to tell me where your line intersects my line.

9. Find the point on $5x + y + z + 17 = 0$ which is closest to $(1,2,3)$.

10. An ant walks along the curve $\vec{r}(t) = t\cos t\,\vec{i} + t\sin t\,\vec{j} + t\,\vec{k}$, for $0 \le t$. Where does the ant touch $x^2 + y^2 + z^2 = 1$?

11. Find the length of the curve $\vec{r}(t) = \frac{t^3}{3}\vec{i} + \frac{t^2}{2}\vec{j}$ for $0 \le t \le 1$.

12. Find the directional derivative of $f(x,y) = x^2 \ln y$ at the point $(1,2)$ in the direction of $\vec{a} = \vec{i} - \vec{j}$.

13. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and is increasing at 3 inches per minute and the radius is 40 inches and is increasing at 2 inches per minute. How fast is the volume increasing at that instant?

14. Find all local maximum points, all local minimum points, and all saddle points of $f(x,y) = x^2y - 6y^2 - 3x^2$.

15. The temperature of a plate at the point (x,y) is $T(x,y) = 20 - x^2 - 2y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(-1,2)$. (The particle always moves in the direction of the greatest increase in temperature.)

16. Find the volume of the solid which is inside both $x^2 + y^2 + z^2 = 16$ and $x^2 + y^2 + (z-6)^2 = 16$ is a solid in $3-$ space.

17. Find $\displaystyle\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$.

18. Consider the solid which is bounded by $2x + 3y + 6z = 12$ and the three coordinate planes. The density of the solid at the point (x,y,z) is x . Find the mass of the solid. Set up the integral, **but do NOT compute the integral.**

19. Find the area inside $r = 6\sin\theta$ and outside $r = 3$.

20. (16 points) Compute $\int_C 2y\, dx + 3x^2\, dy$, where C is the line segment from $(1,2)$ to $(3,4)$.

21. Does there exist a function $f(x,y)$ such that $\overrightarrow{\boldsymbol{\nabla}} f = (6xy+4)\,\overrightarrow{\boldsymbol{i}} + (3x^2-4y)\,\overrightarrow{\boldsymbol{j}}$? If the answer is yes, then find this function $f(x,y)$.

22. Let C be the curve which starts at $(1,0)$; travels along the x-axis until $(2,0)$; travels around the upper part of the circle $x^2+y^2 = 4$ to $(-2,0)$; travels along the x-axis to $(-1,0)$; and finally travels along the upper part of the circle $x^2 + y^2 = 1$ back to $(1,0)$. Compute $\displaystyle\int_C (2x^2 + 6y)\, dx + (3x + 4y^2)\, dy$.