Math 241, Fall 1999, exam 4

1. Let $R = \{(x, y) \mid 1 \le x \le 4,\ 0 \le y \le 2\}$, and let

$$f(x,y) = \begin{cases} 2 & \text{if } 1 \le x < 3,\ 0 \le y \le 2 \\ 3 & \text{if } 3 \le x \le 4,\ 0 \le y \le 2. \end{cases}$$

   Find $\displaystyle\int\int_R f(x,y)\,dA$.

2. Find $\displaystyle\int_0^{\pi/2}\int_0^1 x\sin(xy)\,dy\,dx$.

3. Find $\displaystyle\int_0^{\ln 3}\int_0^1 xye^{xy^2}\,dy\,dx$.

4. Find the volume of the solid in the first octant bounded by $9z = 36 - 9x^2 - 4y^2$ and the coordinate planes.

5. Let $R = \{(x, y) \mid 1 \le x^2 + y^2 \le 4\}$. Find $\displaystyle\int\int_R x^2\,dA$.

6. Find $\displaystyle\int\int_R e^{x^2+y^2}\,dA$, where R is the region enclosed by $x^2 + y^2 = 4$.

7. Find $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}} (4 - x^2 - y^2)^{-1/2}\,dy\,dx$.

8. Find the volume of the solid in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.

9. Find the volume of the solid bounded by the xy plane and $z = 9 - x^2 - y^2$.

10. Find the volume of the solid whose lower bound is $x^2 + y^2 = z$ and whose upper bound is $x^2 + y^2 + z^2 = 12$.