Math 241, Fall 1997, final

PRINT Your Name:_____

There are 19 problems on 8 pages. Problems 1 and 2 are worth 7 points each. Each of the other problems is worth 8 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equation of the plane through $(3,2,1)$, $(2,3,5)$, and $(4,6,7)$.

2. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\overrightarrow{\boldsymbol{a}} = -3\overrightarrow{\boldsymbol{i}} + 4\overrightarrow{\boldsymbol{j}} + 3\overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{b}} = 4\overrightarrow{\boldsymbol{i}} + 3\overrightarrow{\boldsymbol{j}} + 2\overrightarrow{\boldsymbol{k}}$. Find vectors $\overrightarrow{\boldsymbol{u}}$ and $\overrightarrow{\boldsymbol{v}}$ with $\overrightarrow{\boldsymbol{b}} = \overrightarrow{\boldsymbol{u}} + \overrightarrow{\boldsymbol{v}}$, $\overrightarrow{\boldsymbol{u}}$ parallel to $\overrightarrow{\boldsymbol{a}}$, and $\overrightarrow{\boldsymbol{v}}$ perpendicular to $\overrightarrow{\boldsymbol{a}}$.

3. Find the equations of **any** line which is parallel to $2x - 3y + 4z = 12$.

4. Find the equations of the plane tangent to $z = x^2 + y^2$ when $x = 1$ and $y = 2$.

5. Suppose that $z = f(x,y)$, and x and y are writen polar coordinates (that is, $x = r\cos\theta$ and $y = r\sin\theta$). Express $\dfrac{\partial z}{\partial \theta}$ in terms of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

6. An ant walks along the curve $\overrightarrow{\boldsymbol{r}}(t) = t\cos t\,\overrightarrow{\boldsymbol{i}} + t\sin t\,\overrightarrow{\boldsymbol{j}} + t\,\overrightarrow{\boldsymbol{k}}$, for $0 \le t$. Where does the ant touch $x^2 + y^2 + z^2 = 1$?

7. Let $f(x,y) = e^{x\sin(xy)}$. Find $\overrightarrow{\nabla} f$.

8. Let $f(x,y) = \dfrac{y^2}{y^2 + x^4}$. Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along $y = 2x^2$.

9. Find the area inside one loop of $r = 2\cos 4\theta$.

10. Find the point on $(x+13)^2 + (y-6)^2 + (z-1)^2 = 9$ which is closest to $x + 2y + 3z = 100$.

11. Find the area of the region which is bounded by $x + y^2 = 0$ and $2y - x = 3$.

12. Find the absolute extreme points of $f(x,y) = x^2 - y^2 - y$ on $\{(x,y) \mid -2 \le x \le 2,\ -4 \le y \le 2\}$.

13. Find the volume of the solid which is bounded by $z = 18 - x^2 - y^2$ and $z = x^2 + y^2$.

14. Compute $\int_C 2x\,dx - y^3\,dy$ where C is the line segment from $(1,2)$ to $(5,4)$.

15. Compute $\int_C 2y\,dx - 5x\,dy$ where C consists of three line segments. The first line segment for C starts at $(1,3)$ and goes to $(7,3)$; the second segment is from $(7,3)$ to $(4,5)$; and the third segment is from $(4,5)$ to $(1,3)$.

16. Graph $y^2 - x^2 = 1$ in $2-$ space.

17. Graph $y^2 - x^2 = 1$ in $3-$ space.

18. Graph $y^2 - x^2 = z$ in $3-$ space.

19. Graph $y^2 - x^2 = z^2$ in $3-$ space.