Math 241, Fall 1997, exam 4

PRINT Your Name:_____

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Find the equations of the line normal to $z = 3x^2 + 6y^2$ when $x = 1$ and $y = -1$.

2. Suppose that $w = f(x, y, z)$, and x, y, z are writen spherical coordinates (that is, $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, and $z = \rho \cos\phi$). Express $\dfrac{\partial w}{\partial \phi}$ in terms of $\dfrac{\partial w}{\partial x}$ $\dfrac{\partial w}{\partial y}$, and $\dfrac{\partial w}{\partial z}$.

3. Let $f(x, y) = 2xy^2$ and let p be the point $p = (1, 3)$. Find the directional derivative of f at the point p in the direction of $\overrightarrow{\boldsymbol{v}} = 2\overrightarrow{\boldsymbol{i}} + 5\overrightarrow{\boldsymbol{j}}$.

4. Identify all local extreme points and all saddle points of $f(x, y) = 2x^4 - x^2 + 3y^2$.

5. Find the absolute extreme points of $f(x, y) = x^2 - 6x + y^2 - 8y + 7$ on $\{(x, y) \mid x^2 + y^2 \leq 1\}$.

6. Find the minimum of $f(x, y) = x^2 + y^2$ on $xy = 3$.

7. Find $\displaystyle\int_{-1}^{0} \int_{-2}^{2x} e^{y^2}\, dy\, dx$.

8. Find the volume of the solid whose base is $z = 0$ and whose top is $z = 9 - x^2 - y^2$.

9. Consider the solid which is bounded by $x + 3y + 6z = 12$ and the three coordinate planes. The density of the solid at the point (x, y, z) is x. Find the mass of the solid. Set up the integral, **but do NOT compute the integral.**

10. The base of a solid is the region in the $xy-$ plane which is bounded by $y = 4 - x^2$ and $y = x^2 - 4$. The top of the solid is given by $z = x + y + 10$. Find the volume of the solid. Set up the integral, **but do NOT compute the integral.**