Math 241, Fall 1997, exam 3

PRINT Your Name: ______________________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. CIRCLE your answer. NO CALCULATORS! CHECK your answer, whenever possible.

1. Let \( f(x, y) = e^{xy} \sin x + 2xy^2 \). Find \( \nabla f \).

2. Find the equation of the plane tangent to \( z^2 = 3x^2 + 6y^2 \) at \((1, -1, 3)\).

3. Suppose that \( z = f(x, y) \), and \( x \) and \( y \) are written polar coordinates (that is, \( x = r \cos \theta \) and \( y = r \sin \theta \)). Express \( \frac{\partial z}{\partial r} \) in terms of \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \).

4. Let \( f(x, y) = 2x^2 + 5y^3 \) and let \( p \) be the point \( p = (1, 3) \). Find the directional derivative of \( f \) at the point \( p \) in the direction of \( \mathbf{v} = 2 \mathbf{i} + 5 \mathbf{j} \).

5. Where do the lines
\[
\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{1} = \frac{y}{2} = \frac{z+2}{3}
\]
intersect?

6. The temperature of a plate at the point \((x, y)\) is \( T(x, y) = 10 + x^2 - 3y^2 \). Find the path that a heat seeking particle would travel if it starts at the point \((1, 3)\). (The particle always moves in the direction of the greatest increase in temperature.)

7. (There is no partial credit for this problem. Make sure your answer is correct.) Find the equation of the plane through \((2, 2, 1)\), \((1, 2, 3)\), and \((4, 4, 6)\).

8. Let \( f(x, y) = \frac{xy^2}{2x^2 + y^4} \).
   (a) Calculate the limit of \( f(x, y) \) as \((x, y) \to (0, 0)\) along every straight line of the form \( y = mx \).
   (b) Calculate the limit of \( f(x, y) \) as \((x, y) \to (0, 0)\) along the parabola \( x = y^2 \).
   (c) What is \( \lim_{(x,y) \to (0,0)} f(x, y) \)?

9. Find the equations of any line which intersects and is perpendicular to \[
\frac{x-2}{3} = \frac{y-5}{2} = \frac{z-4}{5} \]. Be sure to tell me where your line intersects my line.

10. Graph and label the level sets \( f = 0 \), \( f = 1 \), and \( f = -1 \) for \( f(x, y) = x^2 - y^2 \).