Math 241, Fall 1997, exam 3

PRINT Your Name:____________________________________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Let $f(x,y) = e^{xy}\sin x + 2xy^2$. Find $\overrightarrow{\nabla} f$.

2. Find the equation of the plane tangent to $z^2 = 3x^2 + 6y^2$ at $(1,-1,3)$.

3. Suppose that $z = f(x,y)$, and x and y are writen polar coordinates (that is, $x = r\cos\theta$ and $y = r\sin\theta$). Express $\dfrac{\partial z}{\partial r}$ in terms of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

4. Let $f(x,y) = 2x^2 + 5y^3$ and let p be the point $p = (1,3)$. Find the directional derivative of f at the point p in the direction of $\overrightarrow{\boldsymbol{v}} = 2\overrightarrow{\boldsymbol{i}} + 5\overrightarrow{\boldsymbol{j}}$.

5. Where do the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{1} = \frac{y}{2} = \frac{z+2}{3}$$

intersect?

6. The temperature of a plate at the point (x,y) is $T(x,y) = 10 + x^2 - 3y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(1,3)$. (The particle always moves in the direction of the greatest increase in temperature.)

7. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equation of the plane through $(2,2,1)$, $(1,2,3)$, and $(4,4,6)$.

8. Let $f(x,y) = \dfrac{xy^2}{2x^2 + y^4}$.
   (a) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along every straight line of the form $y = mx$.
   (b) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the parabola $x = y^2$.
   (c) What is $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$?

9. Find the equations of **any** line which **intersects** and is **perpendicular** to $\dfrac{x-2}{3} = \dfrac{y-5}{2} = \dfrac{z-4}{5}$. Be sure to tell me where your line intersects my line.

10. Graph and label the level sets $f = 0$, $f = 1$, and $f = -1$ for $f(x,y) = x^2 - y^2$.