MATH 241, FALL 2001, EXAM 1

PRINT Your Name: ____________________________

There are 10 problems on 4 pages. Each problem is worth 10 points. SHOW your work. [CIRCLE] your answer. NO CALCULATORS!

1. Graph and describe the graph of \( x^2 + y^2 + z^2 = 1 \) in \( 3- \) space.

2. Graph and describe the graph of \( x^2 + y^2 = 1 \) in \( 3- \) space.

3. Graph and describe the graph of \( x^2 + y^2 = 0 \) in \( 3- \) space.

4. Consider the triangle with vertices \( P = (1, 2, 3) \), \( Q = (0, 1, 2) \), and \( R = (2, 4, 7) \). Find the angle of this triangle at the vertex \( Q \).

5. (There is no partial credit for this problem. Make sure your answer is correct.) Let \( \overrightarrow{a} = 2 \overrightarrow{i} - \overrightarrow{j} + 3 \overrightarrow{k} \) and \( \overrightarrow{b} = 4 \overrightarrow{i} - 13 \overrightarrow{j} + 7 \overrightarrow{k} \). Find vectors \( \overrightarrow{u} \) and \( \overrightarrow{v} \) with \( \overrightarrow{b} = \overrightarrow{u} + \overrightarrow{v} \), \( \overrightarrow{u} \) parallel to \( \overrightarrow{a} \), and \( \overrightarrow{v} \) perpendicular to \( \overrightarrow{a} \). (Every number in the answer is an integer. If you have fractions, either you can rid of them or you have made a mistake.)

6. Find the equation of the plane which contains the point \( (1, 3, 2) \) and is perpendicular to the vector \( \overrightarrow{N} = 2 \overrightarrow{i} - 3 \overrightarrow{j} + 1 \overrightarrow{k} \).

7. Find the point on \( (x - 4)^2 + (y - 7)^2 + (z - 8)^2 = 14 \) which is closest to \( x + 2y + 3z = 0 \).

8. Find the equation of the sphere whose center is \( (3, 5, 4) \) and which is tangent to the \( yz \)-plane.

9. Find the work done by the force \( \overrightarrow{F} = 2 \overrightarrow{i} - 5 \overrightarrow{j} + 6 \overrightarrow{k} \) as it moves an object in a straight line from \( P = (3, 2, 4) \) to \( Q = (9, 4, 6) \). Force is measured in pounds. Distance is measured in feet.

10. Find the distance from the point \( (4, 5, 6) \) to the \( x \)-axis.