Math 241, Fall 2000, Final Exam

PRINT Your Name:________________________________

Get your course grade from **TIPS/VIP** late on Tuesday or later.

There are 17 problems on 9 pages. Problems 1, 2, and 7 are each worth 8 points. Each of the other problems is worth 9 points. The exam is worth a total of 150 points. SHOW your work. *CIRCLE* your answer. **NO CALCULATORS!**

1. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equation of the plane through $(2,2,3)$, $(2,0,2)$, and $(5,1,1)$.

2. **(There is no partial credit for this problem. Make sure your answer is correct.)** Find the equations of the line through $(6,4,2)$ and $(3,4,7)$.

3. Graph and name $x^2 - y^2 = 1$ in $2-$ space.

4. Graph and name $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{25} = 1$ in $3-$ space.

5. What are the equations of the line tangent to the curve which is parameterized by $\vec{\boldsymbol{r}}(t) = (3t^3+2)\vec{\boldsymbol{i}} + 6t^2\vec{\boldsymbol{j}} + (4t^3+2t)\vec{\boldsymbol{k}}$ at $t=1$?

6. Find the equation of the plane tangent to the surface $z = x^2 + 3y^3$ at the point where $x=2$ and $y=-2$.

7. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\vec{\boldsymbol{a}} = 2\vec{\boldsymbol{i}} + 4\vec{\boldsymbol{j}} + 6\vec{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = 3\vec{\boldsymbol{i}} + 4\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$. Find vectors $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$ with $\vec{\boldsymbol{b}} = \vec{\boldsymbol{u}} + \vec{\boldsymbol{v}}$, $\vec{\boldsymbol{u}}$ parallel to $\vec{\boldsymbol{a}}$, and $\vec{\boldsymbol{v}}$ perpendicular to $\vec{\boldsymbol{a}}$.

8. Find the point on $5x + y + z + 17 = 0$ which is closest to $(1,2,3)$.

9. An ant walks along the curve $\vec{\boldsymbol{r}}(t) = t\cos t\,\vec{\boldsymbol{i}} + t\sin t\,\vec{\boldsymbol{j}} + t\,\vec{\boldsymbol{k}}$, for $0 \le t$. Where does the ant touch $x^2 + y^2 + z^2 = 1$?

10. Find the length of the curve $\vec{\boldsymbol{r}}(t) = \frac{t^3}{3}\vec{\boldsymbol{i}} + \frac{t^2}{2}\vec{\boldsymbol{j}}$ for $0 \le t \le 1$.

11. Find the directional derivative of $f(x,y) = x^3 \ln y$ at the point $(1,2)$ in the direction of $\vec{\boldsymbol{u}} = \frac{1}{\sqrt{2}}(\vec{\boldsymbol{i}} - \vec{\boldsymbol{j}})$.

12. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 200 inches and is increasing at 4 inches per minute and the radius is 50 inches and is increasing at 3 inches per minute. How fast is the volume increasing at that instant? (The volume of a cone is $\frac{1}{3}\pi r^2 h$.)

13. Find all local maximum points, all local minimum points, and all saddle points of $f(x,y) = x^2y - 6y^2 - 3x^2$.