Math 241, Fall 2000, Exam 4

There are 10 problems on 6 pages. Each problem is worth 10 points. SHOW your work. CIRCLE your answer. NO CALCULATORS!

1. Find \( \int_0^1 \int_0^{3x} x^2 \, dy \, dx \).

2. Find the volume of the solid in the first octant which is bounded by \( y = x^2 \), \( x = 0 \), \( z = 0 \), and \( y + z = 1 \).

3. Let \( R \) be the region \( R = \{(x, y) \mid 2 \leq x \leq 8, \text{ and } 2 \leq y \leq 6\} \). Let \( P \) be the partition of \( R \) into six equal squares by the lines \( x = 4 \), \( x = 6 \), and \( y = 4 \). Approximate \( \iint_R (12 - x - y) \, dA \) by calculating the corresponding Riemann sum \( \sum_{k=1}^{6} f(\bar{x}_k, \bar{y}_k) \Delta A_k \), where \((\bar{x}_k, \bar{y}_k)\) is the center of the \( k^{\text{th}} \) box.

4. Identify all local maximum points, all local minimum points, and all saddle points of \( f(x, y) = xy^2 - 6x^2 - 6xy \).

5. Where does the line normal to \( x^2 + y^2 + 2z^2 = 6 \) at \((1, 2, 1)\) intersect \( 2x + 3y + z = 49 \)?

6. Sand is pouring onto a conical pile in such a way that at a certain instant the height is 100 inches and is increasing at 3 inches per minute and the radius is 40 inches and is increasing at 2 inches per minute. How fast is the volume increasing at that instant? (The volume of a cone is \( V = (1/3)\pi r^2 h \).)