3. Graph and describe the graph of the curve \( \mathbf{r}(t) = \cos t \mathbf{i} + t \mathbf{j} + \sin t \mathbf{k} \) in 3-space.

This is a helix around the y-axis. If you only think about x and z, the graph looks like a circle but \( y \) is growing.

[Diagram of a helix]

4. Find the length of the curve \( \mathbf{r}(t) = t^2 \mathbf{i} - 2t^3 \mathbf{j} + 6t^3 \mathbf{k} \) for \( 0 \leq t \leq 1 \).

Length = \( \int_{0}^{1} \| \mathbf{r}'(t) \| \, dt = \int_{0}^{1} \| 2t \mathbf{i} - 6t^2 \mathbf{j} + 18t^2 \mathbf{k} \| \, dt \)

= \( \int_{0}^{1} \sqrt{4t^2 + 36t^4 + (18t^2)^2} \, dt = \int_{0}^{1} 2t \sqrt{1 + 9t^2 + 81t^4} \, dt \)

\((18^2 = 4 \cdot 9^2) \)

I pull out \( 4t \) as \( t \) from each term.

= \( \int_{0}^{1} 2t \cdot 1 + 90t^2 \, dt = \frac{1}{90} \int_{0}^{1} u^2 \, du = \frac{1}{90} \left( \frac{(91)^{3/2}}{3} - \frac{90^3}{3} \right) \)

\( u = 180t^2 \)
\( du = 180t \, dt \)

= \( \frac{2}{90} \left( (91)^{3/2} - 1 \right) \)