Math 241,   Exam 3,   Fall, 2022

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

(1) What is the equation of the plane tangent to $z = x^2 + y^2$ at the point where $x = 1$ and $y = 3$?

(2) Put $3x^2 + 2y^2 - 2z^2 - 12x - 4y + 12z = 8$ in the form

$$A(x - x_0)^2 + B(y - y_0)^2 + C(z - z_0)^2 = D,$$

where x_0, y_0, z_0, A, B, C, and D are numbers.

(3) Consider the function $f(x, y) = 9x^2 + 4y^2$ and the point $P = (1, 2)$.

    (a) Draw the level set $f(x, y) = c$ which contains the point P.

    (b) Calculate $\overrightarrow{\nabla} f|_P$.

    (c) Draw $\overrightarrow{\nabla} f|_P$ on your answer to (3a) with the tail on P.

    (d) Calculate the directional derivative of the function f at the point P in the direction of the vector $\overrightarrow{v} = \overrightarrow{i} + 2\overrightarrow{j}$

(4) Find all local maximum points, local minimum points, and saddle points of $f(x, y) = (y - 2)x^2 - y^2$.

(5) Find the absolute maximum points and absolute minimum points of $f(x, y) = x^2 + y^2 - 2x$ on the closed triangular region with vertices $(2, 0)$, $(0, 2)$, and $(0, -2)$.