

Math 174, Fall 1998, Final Exam

PRINT Your Name: _____

There are 25 problems on 7 pages. Each problem is worth 6 points.

CIRCLE your answers. **No Calculators.**

1. Is the following argument valid?
All honest people pay their taxes.
Darth is not honest.
 \therefore Darth does not pay his taxes.
2. What is the negation of “The sum of any two irrational numbers is irrational”?
3. Give an example of a function f from the set of integers to the set of integers which is onto, but not one-to-one.
4. True or False. If true, **prove** it. If false, then give a **counterexample**. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are functions, with $g \circ f$ onto, then g is onto.
5. What is the coefficient of x^5 in $(2x + 3)^8$?
6. Prove $\binom{6}{0} + \binom{7}{1} + \binom{8}{2} + \cdots + \binom{6+n}{n} = \binom{7+n}{n}$ for all integers n with $0 \leq n$.
7. Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:
 - a. Socko: Lefty killed Sharky.
 - b. Fats: Muscles didn't kill Sharky.
 - c. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
 - d. Muscles: Lefty didn't kill Sharky.Who did kill Sharky?
8. Write the following sentence in if-then form: “A sufficient condition for Hal's team to win the championship is that it win the rest of its games”.
9. True or False. If true, **prove** it. If false, then give a **counterexample**. For all sets A , B , and C , $A \setminus (B \setminus C) = (A \setminus B) \setminus C$.
10. Solve the recurrence relation $d_k = 2d_{k-1} + 3$, for all integers $k \geq 2$, $d_1 = 2$.
11. A single pair of rabbits (male and female) is born at the beginning of a year. Let r_n equal the number of rabbit pairs alive at the end of month n , for each integer $n \geq 1$, and let $r_0 = 1$. Find a recurrence relation for $r_0, r_1, r_2, r_3, \dots$. Assume the following conditions:
 - (a) Rabbit pairs are not fertile during their first month of life, but thereafter give birth to four new male/female pairs at the end of every month.
 - (b) No deaths occur during the year.

12. Multiple choice. Pick the correct answer then PROVE it is correct. If n is an integer with $n \bmod 3 = 1$, then $\lfloor n/3 \rfloor$ is equal to

$$(a) (n+1)/3, \quad (b) n/3, \quad (c) (n-1)/3, \quad (d) (n-2)/3.$$

13. Write 46 in base 16.

14. A coin is tossed 10 times. What is the probability that exactly 6 of the tosses will land as heads?

15. Are $p \wedge (q \vee r)$ and $(p \wedge q) \vee r$ logically equivalent? Justify your answer.

16. Prove that

$$\left(1 - \frac{1}{2^2}\right) \cdot \left(1 - \frac{1}{3^2}\right) \cdot \dots \cdot \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n},$$

for all integers $n \geq 2$.

17. True or False. If true, **prove** it. If false, then give a **counterexample**. If an integer is a perfect square, then its cube root is irrational.

18. True or False. If true, **prove** it. If false, then give a **counterexample**. The difference of any two irrational numbers is irrational.

19. Find integers q and r so that $56 = 5q + r$ with $0 \leq r < 5$.

20. True or False. If true, **prove** it. If false, then give a **counterexample**. The sum of any three consecutive integers is divisible by 3.

21. True or False. If true, **prove** it. If false, then give a **counterexample**. If a sum of two integers is even, then one of the summands is even. (In the expression $a + b$, a and b are called *summands*.)

22. How many solutions does $x_1 + x_2 + x_3 + x_4 = 20$ have, if each x_i is a positive integer?

23. A computer programming team has 25 members: 15 of the members are women and 10 of the members are men. How many ways can a group of 7 be chosen to work on a project if at most 3 women are in the group?

24. How many ways can the letters of the word DESIGN be arranged in a row?

25. Let $A = \{x, y, z, w\}$ and $B = \{a, b\}$. List the elements of $B \times A$.