PRINT Your Name:________________________________
There are 11 problems on 4 pages. The exam is worth 50 points.
$\boxed{CIRCLE}$ your answers. **No Calculators.**

WHEN YOU DO SOMETHING CLEVER, EXPLAIN YOUR WORK.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office door later today, you may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website at about 12:30 today.

You may leave the binbomial coefficient $\binom{n}{r}$ in any of your answers.

1. (5 points) Flip a coin ten times, what is the proabability that the coin lands "Heads" exactly four times.

2. (5 points) A committee consists of 10 people, 6 women and 4 men.
   (a) How many subcommittees which consist of 6 people can be made?
   (b) How many subcommittees which consist of 4 women and 2 men can be made?

3. (4 points) Six friends have pictures made. Each picture consists of 4 people arranged in a straight line? How many arrangements are possible?

4. (4 points) How many integers between 1 and 1000 are relatively prime to 10 ?

5. (5 points) Let $A = \{p, q, r, s\}$ and let S_1 be the set of all subsets of A that do not contain p and S_2 the set of all subsets of A that do contain p .
   (a) Find S_1 .
   (b) Find S_2 .

6. (4 points) True or False. If true, **prove** it. If false, then give a **counterexample**. For all sets A , B , and C , if $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.

7. (5 points) True or False. If true, **prove** it. If false, then give a **counterexample**. For all integers $n \geq 2$,
$$\left(1 - \frac{1}{2^2}\right) \cdot \left(1 - \frac{1}{3^2}\right) \cdot \dots \cdot \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

8. (5 points) True or False. If true, **prove** it. If false, then give a **counterexample**. For all integers $n \geq 1$,
$$1 + 2^4 + 3^4 + \dots + n^4 = n^4 + n - 1.$$

9. (4 points) True or False. If true, **prove** it. If false, then give a **counterexample**. If $p_1, p_2, p_3, \dots, p_r$ are prime integers, then $N = p_1 p_2 p_3 \cdots p_r + 1$ is a prime integer.

10. (4 points) Prove that there is no greatest even integer.

11. (5 points) How many permutations of a, b, c, d, e, f are there in which the first letter is a , b , c , or d and the last letter is c , d , e , or f .