

Solutions to Exam 1, Fall 2003, Math 174

PRINT Your Name: _____

There are 10 problems on 4 pages. Each problem is worth 5 points.

CIRCLE your answers.

1. (a) Write 273 in base 2.

$$273 = 256 + 16 + 1 = 2^8 + 2^4 + 2^0 = \boxed{100010001_2}.$$

- (b) Write 273 in base 16.

$$273 = 256 + 16 + 1 = 16^2 + 16^1 + 1 = \boxed{111_{16}}.$$

2. Are $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ logically equivalent? Justify your answer. These two statements ARE logically equivalent. This is the formula for distribution. Here is a truth table

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

The two columns on the right have the same truth values for all choices for p , q , r . Thus, $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ ARE logically equivalent.

3. What is negation of $x < 2$ or $4 \leq x$?

$$\boxed{2 \leq x < 4}.$$

4. Write $(p \vee \sim q) \rightarrow r$ using \wedge , \vee , and \sim , but not \rightarrow .

$(p \vee \sim q) \rightarrow r$ is logically equivalent to $\sim(p \vee \sim q) \vee r$.

5. Is the argument

$$\begin{array}{l} p \rightarrow q \\ \sim p \\ \therefore \sim q \end{array}$$

valid? Justify your answer.

This argument is NOT valid. It exhibits the inverse error. Here is a truth table.

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	
T	T	T	F	F	
T	F	F	F	T	
F	T	T	T	F	★
F	F	T	T	T	

The hypotheses are both true in rows 3 and 4. The conclusion is false in row 3. This argument is not valid.

6. True or False. If true, **prove** it. If false, then give a **counterexample**. For all integers n and m , if $n - m$ is even, then $n^3 - m^3$ is even.

TRUE. Assume $n - m$ is even. We will prove that $n^3 - m^3$ is also even. We know that $n - m = 2\ell$ for some integer ℓ . It follows that

$$n^3 - m^3 = (n - m)(n^2 + nm + m^2) = 2\ell(n^2 + nm + m^2).$$

The number $\ell(n^2 + nm + m^2)$ is an integer; hence, $n^3 - m^3$ is also even.

7. True or False. If true, **prove** it. If false, then give a **counterexample**. The sum of any two irrational numbers is irrational.

This statement is FALSE. The numbers $\sqrt{2}$ and $-\sqrt{2}$ are both irrational, but the sum of these two numbers is zero, which is rational.

8. Is the argument:

All healthy people eat an apple a day.

Helen eats an apple a day;

therefore, Helen is a healthy person.

valid? Justify your answer.

This argument is NOT valid. It exhibits the converse error. Let p represent Helen is healthy, and q represent Helen eats an apple a day. The argument is

$$\begin{array}{c} p \rightarrow q \\ q \\ \therefore p \end{array}$$

If you need a truth table, it is

p	q	$p \rightarrow q$	q	p	
T	T	T	T	T	
T	F	F	F	T	
F	T	T	T	F	★
F	F	T	F	F	

The hypotheses are all true in rows 1 and 3. The conclusion is false in row 3. The argument is not valid.

9. Write the following sentence in if – then form: “Earning a grade of C minus in this course is a sufficient condition for it to count toward graduation.”

If one earns a grade of C minus in this course, then the course counts towards graduation.

10. What is the negation of: \forall colors C , \exists an animal A such that A is colored C .

\exists a color C , such that \forall animals A , A is not colored C .