16. Prove that
\[
(1 - \frac{1}{2^2}) \cdot (1 - \frac{1}{3^2}) \cdot \ldots \cdot (1 - \frac{1}{n^2}) = \frac{n+1}{2n},
\]
for all integers \( n \geq 2 \).

\[\text{LHS} = \frac{3}{4} \quad \text{RHS} = \frac{3}{4} \quad \checkmark\]

Assume \( I H \) \( (1 - \frac{1}{2^2}) \cdot (1 - \frac{1}{3^2}) \cdot \ldots \cdot (1 - \frac{1}{n^2}) = \frac{n+1}{2n} \)

We next prove \( (1 - \frac{1}{2^2}) \cdot (1 - \frac{1}{3^2}) \cdot \ldots \cdot (1 - \frac{1}{n^2}) \cdot (1 - \frac{1}{(n+1)^2}) = \frac{n+2}{2n+2} \).

\[\text{LHS} = \frac{n+1}{2n} \cdot \frac{n+1}{2n+2} = \frac{(n+1)^2}{4n(n+1)} = \frac{n+1}{2n(n+1)} \quad \checkmark \]

which equals the right side.

17. True or False. If true, prove it. If false, then give a counterexample. If an integer is a perfect square, then its cube root is irrational.

False 64 is a perfect square and \( \sqrt[3]{64} = 4 \) which is rational.

18. True or False. If true, prove it. If false, then give a counterexample. The difference of any two irrational numbers is irrational.

False \( \sqrt{2} \) and \( \sqrt{3} \) are both irrational but \( \sqrt{2} - \sqrt{3} = 0 \) which is rational.

19. Find integers \( q \) and \( r \) so that \( 56 = 5q + r \) with \( 0 \leq r < 5 \).

\( \boxed{q = 11, r = 1} \)