PRINT Your Name: ____________________________
There are 12 problems on 6 pages. Four of the problems are worth 9 points. Each of the other problems is worth 8 points. **CIRCLE** your answers. **No Calculators.** Show your work.

1. True or False. If true, prove it. If false, then give a counterexample. If \( f : X \to Y \) and \( g : Y \to Z \) are functions, with \( g \circ f \) onto, then \( f \) is onto.

\( \text{False} \)

Consider \( X \xrightarrow{f} Y \xrightarrow{g} Z \)

\( g \circ f \) is onto but \( f \) is not onto.

2. True or False. **Prove** your answer. The sets \( S = \{ x \in \mathbb{R} \mid 0 < x < 1 \} \) and \( U = \{ x \in \mathbb{R} \mid 0 < x < 2 \} \) have the same cardinality.

\( \text{True} \)

Consider \( f : S \to U \) given by \( f(x) = 2x \)

We see that \( f \) is onto. If \( y \in U \) then \( 0 < y < 2 \) so \( 0 < \frac{y}{2} < 1 \).

So \( \frac{y}{2} \in S \) and \( f(\frac{y}{2}) = y \).

We see that \( f \) is 1-1: If \( f(x) = f(y) \) then \( 2x = 2y \) so \( x = y \).