The vat shown in the accompanying figure contains water to a depth of 2 m. Find the work required to pump all the water to the top of the vat. [Use 9810 N/m³ as the weight density of water.]

Notice how I have oriented my axes. I put x=0 at the top of the vat. I will calculate the work required to lift the thin layer of water whose x-coordinate is x. I will do this for each x from x = 1 to x = 3. The work to lift the water at x-coordinate x is:

\[ \text{Vol of water} \cdot \text{Density of water} \cdot \text{distance lifted} \]

\[ = W \cdot L \cdot x \cdot 9810 \frac{N}{m^3} \cdot x \]

\[ L = dx \quad L = 6m \quad \text{use similar triangle to see} \]

\[ \frac{w}{2} = \frac{3-x}{3} \quad \text{or} \quad w = \frac{4}{3} (3-x) \]

The total work done is:

\[ \int_{1}^{3} \left( \frac{4}{3} (3-x) \right) 6 (9810) x \, dx \]

\[ = 8 (9810) \left[ \frac{3}{2} (3-x)^2 \right]_{1}^{3} \]

\[ = 9 (9810) \left( \frac{27}{2} - 9 - \frac{3}{2} + \frac{1}{3} \right) = \left( \frac{8 (9810) (10)}{3} \right) J. \]

Feb. 2 Quiz