Find the area between \( y = \frac{2}{1+x^2} \) and \( y = |x| \).

**Answer:** Draw the picture. We see that \( \frac{2}{1+x^2} \) is above \( |x| \). The two graphs intersect when \( |x| = \frac{2}{1+x^2} \). The picture is symmetric across the \( y \)-axis, so we may as well take \( x > 0 \). The graphs intersect when \( x = \frac{2}{1+x^2} \). So, \( x(1+x^2) = 2 \); that is, \( x^3 + x - 2 = 0 \). It is clear that \( x = 1 \) is a solution of the equation. (There aren't any other real solutions because \( x^3 + x - 2 = (x - 1)(x^2 + x + 2) \) and \( x^2 + x + 2 \) has no real roots.) I will take advantage of the symmetry and double the area inside the part of the picture in the first quadrant. The answer is

\[
2 \int_0^1 \left( \frac{2}{1+x^2} - x \right) dx = 2 \left( 2 \arctan x - \frac{x^2}{2} \right) \bigg|_0^1 = 2 \left( 2 \frac{\pi}{4} - \frac{1}{2} \right) = \pi - 1.
\]