PRINT Your Name:__________________

Quiz – January 17, 2006

Find the area between $y = \frac{2}{1+x^2}$ and $y = |x|$.

Answer: Draw the picture. We see that $\frac{2}{1+x^2}$ is above $|x|$. The two graphs intersect when $|x| = \frac{2}{1+x^2}$. The picture is symmetric across the y-axis, so we may as well take $x > 0$. The graphs intersect when $x = \frac{2}{1+x^2}$. So, $x(1+x^2) = 2$; that is, $x^3 + x - 2 = 0$. It is clear that $x = 1$ is a solution of the equation. (There aren't any other real solutions because $x^3 + x - 2 = (x-1)(x^2+x+2)$ and $x^2 + x + 2$ has no real roots.) I will take advantage of the symmetry and double the area inside the part of the picture in the first quadrant. The answer is

$$2\int_0^1 \left(\frac{2}{1+x^2} - x\right) dx = 2\left(2\arctan x - \frac{x^2}{2}\right)\Bigg|_0^1 = 2(2\frac{\pi}{4} - \frac{1}{2}) = \boxed{\pi - 1}.$$