

PRINT Your Name: _____

Quiz 10 — March 23, 2012 – Section 7 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Write in complete sentences.

The quiz is worth 5 points.

Approximate the sum $\sum_{n=1}^{\infty} \frac{1}{n^5}$ with an error at most 5×10^{-4} . Explain your work very thoroughly. Write in complete sentences.

Answer: We approximate $\sum_{n=1}^{\infty} \frac{1}{n^5}$ with $\sum_{n=1}^N \frac{1}{n^5}$ for some fixed, but not yet determined, integer N . We use the picture which is drawn below to see that

$$\left| \sum_{n=1}^{\infty} \frac{1}{n^5} - \sum_{n=1}^N \frac{1}{n^5} \right| = \sum_{n=N+1}^{\infty} \frac{1}{n^5} = \text{the area inside the boxes}$$

$$\leq \text{the area under the curve} = \int_N^{\infty} x^{-5} dx = \lim_{b \rightarrow \infty} \left. \frac{-1}{4x^4} \right|_N^b = \lim_{b \rightarrow \infty} \frac{-1}{4b^4} + \frac{1}{4N^4} = \frac{1}{4N^4}.$$

We make

$$\frac{1}{4N^4} \leq 5 \times 10^{-4}.$$

We make

$$\frac{10^4}{20} \leq N^4.$$

We make

$$500 \leq N^4.$$

We see that $4^4 < 500 < 5^4$; so we take N to be 5. We conclude that:

$\sum_{n=1}^5 \frac{1}{n^5} \text{ approximates } \sum_{n=1}^{\infty} \frac{1}{n^5} \text{ with an error at most } 5 \times 10^{-4}.$

