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Quiz – September 26, 2006

Find $\int \tan^3 4x dx$.

Answer: Save one $\tan 4x$. Use $\tan^2 \theta + 1 = \sec^2 \theta$ to convert the other two $\tan 4x$ into secants. The problem is

$$\int \tan 4x (\sec^2 4x - 1) dx = \int \tan 4x \sec^2 4x dx - \int \frac{\sin 4x}{\cos 4x} dx.$$

In the first integral, let $u = \tan 4x$, so $du = 4 \sec^2 4x dx$. In the second integral, let $w = \cos 4x$, so $dw = -4 \sin 4x dx$. The original integral is equal to

$$\frac{1}{4} \int u du + \frac{1}{4} \int \frac{1}{w} dw = \frac{1}{4} \frac{u^2}{2} + \frac{1}{4} \ln |w| + C = \boxed{\frac{1}{4} \frac{\tan^2 4x}{2} + \frac{1}{4} \ln |\cos 4x| + C}.$$

Check: The derivative of the proposed answer is

$$\frac{1}{4} \tan 4x (\sec^2 4x) 4 + \frac{1}{4} \frac{(-\sin 4x)}{\cos 4x} 4 = \tan 4x (\sec^2 4x - 1) = \tan^3 4x. \checkmark$$