PRINT Your Name:______________________________________

Quiz – September 24, 2004

Find

$$\int \sqrt{5-4x-x^2}\,dx.$$

Check your answer if you have time and energy.

Answer: Complete the square:

$$5-4x-x^2 = 5+4-(x^2+4x+4)$$

to see that the original problem is equal to

$$\int \sqrt{9-(x+2)^2}\,dx.$$

I find the expression a^2-u^2 to be ugly in this problem, so I let $u=a\sin\theta$; that is, I let $x+2=3\sin\theta$. It follows that $dx=3\cos\theta d\theta$. Notice that

$$\sqrt{9-(x+2)^2}=\sqrt{9-9\sin^2\theta}=3\cos\theta.$$

The original problem is

$$9\int\cos^2\theta\,d\theta=\frac{9}{2}\int 1+\cos 2\theta d\theta=\frac{9}{2}\left(\theta+\frac{\sin 2\theta}{2}\right)+C$$

$$=\frac{9}{2}\left(\theta+\frac{2\sin\theta\cos\theta}{2}\right)+C=\boxed{\frac{9}{2}\left(\arcsin\left(\frac{x+2}{3}\right)+\frac{(x+2)\sqrt{9-(x+2)^2}}{9}\right)+C.}$$

Check: The derivative of the proposed answer is

$$\frac{9}{2}\left(\frac{1}{3\sqrt{1-\frac{(x+2)^2}{9}}}+\frac{1}{9}\left((x+2)\frac{-2(x+2)}{2\sqrt{9-(x+2)^2}}+\sqrt{9-(x+2)^2}\right)\right)$$

$$=\frac{9}{2}\left(\frac{1}{3\sqrt{\frac{9-(x+2)^2}{9}}}+\frac{1}{9}\left(\frac{-(x+2)^2}{\sqrt{9-(x+2)^2}}+\frac{9-(x+2)^2}{\sqrt{9-(x+2)^2}}\right)\right)$$

$$=\frac{9}{2}\left(\frac{1}{\sqrt{9-(x+2)^2}}+\frac{9-2(x+2)^2}{9\sqrt{9-(x+2)^2}}\right)$$

$$=\frac{9}{2}\left(\frac{9}{9\sqrt{9-(x+2)^2}}+\frac{9-2(x+2)^2}{9\sqrt{9-(x+2)^2}}\right)$$

$$=\frac{9}{2}\left(\frac{2(9-(x+2)^2)}{9\sqrt{9-(x+2)^2}}\right)$$

$$=\frac{9-(x+2)^2}{\sqrt{9-(x+2)^2}}=\sqrt{9-(x+2)^2}=\sqrt{9-(x^2+4x+4)}=\sqrt{5-4x-x^2}.\checkmark$$