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Quiz – April 8, 2004

Consider the power series

$$f(x) = 1 + \frac{x+1}{2} + \frac{(x+1)^2}{2^2} + \frac{(x+1)^3}{2^3} + \dots .$$

For which x does $f(x)$ converge? Justify your answer.

Answer: The series is

$$f(x) = \sum_{k=0}^{\infty} \frac{(x+1)^k}{2^k}.$$

We apply the ratio test. Let

$$\rho = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k\to\infty} \left| \frac{\frac{(x+1)^{k+1}}{2^{k+1}}}{\frac{(x+1)^k}{2^k}} \right| = \lim_{k\to\infty} \left| \frac{(x+1)^{k+1}}{2^{k+1}} \frac{2^k}{(x+1)^k} \right| = \lim_{k\to\infty} \frac{|x+1|}{2}.$$

If $\rho < 1$, then $f(x)$ converges. If $1 < \rho$, then $f(x)$ diverges. We have to study $\rho = 1$ further. Well, $\rho < 1$ when $\frac{|x+1|}{2} < 1$; that is $|x+1| < 2$; that is, $-2 < x+1 < 2$; or $-3 < x < 1$. We also see that $1 < \rho$ for $x < -3$ also for $1 < x$. The endpoints $x = -3$ and $x = 1$ need special attention. We see that

$$f(-3) = \sum_{k=0}^{\infty} \frac{(-2)^k}{2^k} = \sum_{k=0}^{\infty} \frac{(-1)^k 2^k}{2^k} = \sum_{k=0}^{\infty} (-1)^k$$

which diverges by the Individual Term Test for Divergence since $\lim_{k\to\infty} (-1)^k$ does not exist; and hence is not equal to zero. Also,

$$f(1) = \sum_{k=0}^{\infty} \frac{2^k}{2^k} = \sum_{k=0}^{\infty} 1$$

which diverges by the Individual Term Test for Divergence since $\lim_{k\to\infty} 1 = 1$, which is not zero. We conclude that

$$\boxed{f(x) \text{ converges for } -3 < x < 1 \text{ and diverges everywhere else.}}$$