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Quiz – April 6, 2004

Is the series

$$\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}$$

absolutely convergent, conditionally convergent, or divergent? Justify your answer.

Answer: The series of absolute values

$$\sum_{n=2}^{\infty}\frac{1}{n\ln n}$$

diverges by the integral test. Notice that the function $f(x)=\frac{1}{x\ln x}$ is positive and decreasing for $2\le x$. Also, the integral

$$\int_2^{\infty}\frac{1}{x\ln x}dx=\lim_{b\to\infty}\ln|\ln x|\Big|_2^b=\lim_{b\to\infty}\ln|\ln b|-\ln(\ln 2)=+\infty.$$

The integral $\int_2^{\infty}\frac{1}{x\ln x}dx$ diverges. So, the series

$$\sum_{n=2}^{\infty}\frac{1}{n\ln n}$$

also diverges.

We apply the alternating series test to the given series

$$\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}.$$

Notice that the terms alternate in sign. The terms, in absolute value, are decreasing (since the larger n is the smaller $\frac{1}{n\ln n}$ is). The terms go to zero since $\lim\limits_{n\to\infty}\frac{1}{n\ln n}=0$. The alternating series test shows that

$$\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}$$

converges. We conclude that

$$\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}$$

is conditionally convergent.