PRINT Your Name:

What familiar function is equal to

$$f(x) = x - x^2 + x^3 - x^4 + x^5 - \dots$$
?

Justify your answer.

Answer: Recall that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$$

for -1 < x < 1. Replace x by -x to see that

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots$$

for -1 < -x < 1. (Of course -1 < -x < 1 if and only if -1 < x < 1.) Multiply by x to see that

$$\boxed{\frac{x}{1+x} = x - x^2 + x^3 - x^4 + x^5 - x^6 + \dots}$$