Quiz – April 13, 2004

What familiar function is equal to

\[ f(x) = x - x^2 + x^3 - x^4 + x^5 - \ldots \]?

Justify your answer.

**Answer:** Recall that

\[ \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots \]

for \(-1 < x < 1\). Replace \(x\) by \(-x\) to see that

\[ \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots \]

for \(-1 < -x < 1\). (Of course \(-1 < -x < 1\) if and only if \(-1 < x < 1\).) Multiply by \(x\) to see that

\[ \frac{x}{1+x} = x - x^2 + x^3 - x^4 + x^5 - x^6 + \ldots \].