Quiz – February 5, 2004

Find
\[ \int \frac{t^2 \cos^2(t^3 - 2)}{\sin^2(t^3 - 2)} \, dt. \]

Check your answer.

**Answer:** Let \( u = t^3 - 2 \). So, \( du = 3t^2 \, dt \), and \( \frac{1}{3} \, du = t^2 \, dt \). The original integral is equal to
\[
\frac{1}{3} \int \frac{\cos^2 u}{\sin^2 u} \, du = \frac{1}{3} \int \cot^2 u \, du.
\]

Recall that \( \sin^2 u + \cos^2 u = 1 \). Divide both sides by \( \sin^2 u \) to see that \( 1 + \cot^2 u = \csc^2 u \). The original problem is
\[
= \frac{1}{3} \int (\csc^2 u - 1) \, du = \frac{1}{3} (-\cot u - u) + C = \frac{1}{3} (\cot(t^3 - 2) + (t^3 - 2)) + C.
\]

**Check:** The derivative of the proposed answer is
\[
-\frac{1}{3} [3t^2(-\csc^2(t^3 - 2) + 3t^2)] = t^2(\csc^2(t^3 - 2) - 1) = t^2 \cot^2(t^3 - 2). \checkmark
\]