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**Quiz – February 10, 2004**

Find

$$\int \sin^5 4x \cos^2 4x \, dx.$$

Check your answer.

**Answer:** The original problem is equal to

$$\int \sin 4x (1 - \cos^2 4x)^2 \cos^2 4x \, dx.$$

Let  $u = \cos 4x$ . So,  $du = -4 \sin 4x \, dx$ . The original integral is equal to

$$\begin{aligned} -\frac{1}{4} \int (1 - u^2)^2 u^2 \, du &= -\frac{1}{4} \int (1 - 2u^2 + u^4) u^2 \, du = -\frac{1}{4} \int (u^2 - 2u^4 + u^6) \, du \\ &= -\frac{1}{4} \left( \frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} \right) + C = \boxed{-\frac{1}{4} \left( \frac{\cos^3 4x}{3} - \frac{2 \cos^5 4x}{5} + \frac{\cos^7 4x}{7} \right) + C.} \end{aligned}$$

**Check:** The derivative of the proposed answer is

$$\begin{aligned} -\frac{1}{4} (\cos^2 4x (-4 \sin 4x) - 2 \cos^4 4x (-4 \sin 4x) + \cos^6 4x (-4 \sin 4x)) \\ &= -\frac{-4 \sin 4x}{4} \cos^2 4x (1 - 2 \cos^2 4x + \cos^4 4x) \\ &= \sin 4x \cos^2 4x (1 - \cos^2 4x)^2 \\ &= \sin 4x \cos^2 4x (\sin^2 4x)^2 = \sin^5 4x \cos^2 4x. \checkmark \end{aligned}$$