PRINT Your Name:____________________

Quiz – November 2, 2006

Find all values of x for which the series converges and find the sum of the series for those values of x. The series is

$$\frac{1}{x^2}+\frac{2}{x^3}+\frac{4}{x^4}+\frac{8}{x^5}+\frac{16}{x^6}+\dots\ .$$

Answer: The series is the geometric series with ratio $r=\frac{2}{x}$ and initial term $a=\frac{1}{x^2}$. If $-1<\frac{2}{x}<1$, then the series converges to

$$\frac{a}{1-r}=\frac{\frac{1}{x^2}}{1-\frac{2}{x}}=\frac{1}{x^2-2x}.$$

The expression "$-1<\frac{2}{x}<1$" means that $-1<\frac{2}{x}$ AND at the very same time $\frac{2}{x}<1$. In particular, if $0<x$, then "$-1<\frac{2}{x}<1$" is exactly the same as $2<x$. On the other hand, if $x<0$, then "$-1<\frac{2}{x}<1$" is exactly the same as $x<-2$. We conclude that:

$$\boxed{\begin{array}{c}\frac{1}{x^2}+\frac{2}{x^3}+\frac{4}{x^4}+\frac{8}{x^5}+\frac{16}{x^6}+\dots \text{ converges if } x<-2 \text{ OR if } 2<x.\\ \text{Furthermore, in these cases, the sum of the series is } \frac{1}{x^2-2x}.\end{array}}$$