

PRINT Your Name: _____

Quiz – October 5, 2006

Find $\int \frac{2x^2 + 3}{x(x-1)^2} dx$.

Answer: Use the method of partial fractions. Set

$$\frac{2x^2 + 3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}.$$

Multiply both sides by $x(x-1)^2$ to get

$$2x^2 + 3 = A(x-1)^2 + Bx(x-1) + Cx.$$

Clean this up to be

$$\begin{array}{rcl} 2x^2 & +3 & = & Ax^2 & -2Ax & +A \\ & & & +Bx^2 & -Bx & \\ & & & & Cx & \end{array}$$

Equate the corresponding coefficients to see that

$$2 = A + B, \quad 0 = -2A - B + C, \quad 3 = A.$$

So, $A = 3$, $B = -1$, and $C = 5$. The original problem is

$$\int \frac{3}{x} + \frac{-1}{x-1} + \frac{5}{(x-1)^2} dx = \boxed{3 \ln|x| - \ln|x-1| - \frac{5}{x-1} + C}.$$

Check The derivative of my proposed answer is:

$$\begin{aligned} \frac{3}{x} - \frac{1}{x-1} + \frac{5}{(x-1)^2} &= \frac{3(x-1)^2 - x(x-1) + 5x}{x(x-1)^2} \\ &= \frac{3x^2 - 6x + 3 - x^2 + x + 5x}{x(x-1)^2} = \frac{2x^2 + 3}{x(x-1)^2}. \quad \checkmark \end{aligned}$$