Quiz – October 5, 2004

Find

\[ \int \frac{x + 1}{(x - 3)^2} \, dx. \]

Check your answer.

**Answer:** Multiply both sides of

\[ \frac{x + 1}{(x - 3)^2} = \frac{A}{x - 3} + \frac{B}{(x - 3)^2} \]

by \((x - 3)^2\) to get:

\[ x + 1 = A(x - 3) + B; \]

so,

\[ x + 1 = Ax + (-3A + B). \]

Equate the coefficients to see that

\[ 1 = A, \quad 1 = -3A + B. \]

So, \(A = 1\) and \(B = 4\). We check that

\[ \frac{1}{x - 3} + \frac{4}{(x - 3)^2} = \frac{(x - 3) + 4}{(x - 3)^2} = \frac{x + 1}{(x - 3)^2} \]

So, the original problem is equal to

\[ \int \frac{1}{x - 3} + \frac{4}{(x - 3)^2} \, dx = \ln |x - 3| - \frac{4}{x - 3} + C. \]

**Check:** The derivative of the proposed answer is

\[ \frac{1}{x - 3} + \frac{4}{(x - 3)^2} \]

and we have already seen that this is \(\frac{x + 1}{(x - 3)^2}\).