PRINT Your Name:____________________________________

Quiz – October 29, 2004

Does the series $\sum_{k=1}^{\infty} \frac{2}{(k+2)k}$ converge or diverge? If the series converges, find its sum.

Answer: If one thinks to try partial fractions on the fraction $\frac{2}{(k+2)k}$, then one can see that this series is a telecoping series and thereby find the sum of the series. Multiply

$$\frac{2}{(k+2)k} = \frac{A}{k+2} + \frac{B}{k}$$

by $(k+2)k$ to see that

$$2 = Ak + B(k+2)$$

or

$$2 = (A+B)k + 2B.$$

Equate the corresponding coefficients to see that $B = 1$ and $A = -1$. Thus, the series is

$$\sum_{k=1}^{\infty} \left[\frac{1}{k} - \frac{1}{k+2}\right].$$

The n^{th} partial sum is

$$s_n = \sum_{k=1}^{n} \left[\frac{1}{k} - \frac{1}{k+2}\right]$$

$$= \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{n-2} - \frac{1}{n}\right) + \left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right)$$

$$= \frac{1}{1} + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}.$$

The sum of the series is the limit of the sequence of partial sums. So the sum of the series is

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{1} + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} = \frac{3}{2}.$$

We conclude that the series converges to $\frac{3}{2}$.