Let \( a_n \) be the average value of \( f(x) = \frac{1}{x} \) over the interval \([1, n]\). Determine whether the sequence \( \{a_n\} \) converges, and, if so, find its limit.

**Answer:** We see that

\[
a_n = \frac{1}{n-1} \int_1^n \frac{1}{x} \, dx = \frac{1}{n-1} \ln x \bigg|_1^n = \frac{1}{n-1} \ln n.
\]

It follows that

\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{\ln n}{n-1}.
\]

The top and the bottom both go to infty. We use L’hopital’s rule to see that

\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{1}{1} = 0.
\]

We conclude that the sequence \( \{a_n\} \) converges to 0.