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Quiz – October 17, 2006

Let a_n be the average value of $f(x) = \frac{1}{x}$ over the interval $[1, n]$. Determine whether the sequence $\{a_n\}$ converges, and, if so, find its limit.

Answer: We see that

$$a_n = \frac{1}{n-1}\int_1^n \frac{1}{x}dx = \frac{1}{n-1}\ln x\Big|_1^n = \frac{1}{n-1}\ln n.$$

It follows that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{\ln n}{n-1}.$$

The top and the bottom both go to infty. We use L'hopital's rule to see that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{\frac{1}{n}}{1} = 0.$$

We conclude that the sequence $\{a_n\}$ converges to 0.