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Quiz – October 10, 2006

Find $\displaystyle\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}dx$.

Answer: Use the method of partial fractions. Set

$$\frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}.$$

Multiply both sides by $(x^2+1)(x^2+3)$ to get

$$x^3+3x^2+x+9 = (Ax+B)(x^2+3) + (Cx+D)(x^2+1).$$

Clean this up to be

$$\begin{array}{rcccc} x^3+3x^2+x+9 = & Ax^3 & & +3Ax & \\ & & +Bx^2 & & +3B \\ & +Cx^3 & & +Cx & \\ & & +Dx^2 & & +D \end{array}$$

Equate the corresponding coefficients to see that

$$1 = A + C, \quad 3 = B + D, \quad 1 = 3A + C, \quad 9 = 3B + D.$$

Subtract $1 = 3A + C$ minus $1 = A + C$ to see $0 = 2A$ or $0 = A$ and therefore $C = 1$. Subtract $9 = 3B + D$ minus $3 = B + D$ to see $6 = 2B$ or $3 = B$ and therefore $D = 0$. At this point we claim that

$$\frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)} = \frac{3}{x^2+1} + \frac{x}{x^2+3}.$$

Let us check this much before we go any further. The right side is

$$\frac{3x^2+9+x^3+x}{(x^2+1)(x^2+3)}$$

as we expected. So, the original problem is

$$\int \frac{3}{x^2+1} + \frac{x}{x^2+3}dx = \boxed{3\arctan x + \tfrac{1}{2}\ln(x^2+3) + C.}$$