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Quiz – January 15, 2004

Let $f(x) = \dfrac{x-1}{x+1}$ for all x, except $x=-1$.

(a) Find a formula for $f^{-1}(x)$.
(b) What is the domain of $f^{-1}(x)$?
(c) Verify that $f^{-1}(f(x)) = x$ for all x in the domain of f.

Answer: Let $y = f^{-1}(x)$. Our job is to find y. We know that $f(y) = f(f^{-1}(x)) = x$. So, y is in the domain of f and $\frac{y-1}{y+1} = x$. Multiply both sides by $y+1$ to get $y-1 = x(y+1)$; hence, $y - xy = x+1$, or $y(1-x) = x+1$. Thus, $y = \frac{x+1}{1-x}$. So the answer to (a) is

$$\boxed{f^{-1}(x) = \frac{x+1}{1-x}.}$$

(b) The domain of $f^{-1}(x)$ is every x, except $x=1$.
(c) We see that

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1}+1}{1-\frac{x-1}{x+1}}.$$

Multiply top and bottom by $x+1$ to see that

$$f^{-1}(f(x)) = \frac{x-1+x+1}{x+1-(x-1)} = \frac{2x}{2} = x.\checkmark$$