PRINT Your Name:____________________

Quiz 8 — October 17, 2012 – Section 9 – 10:10 – 11:00

Remove everything from your desk except a pencil or pen.

Circle your answer. **Show your work.** Your work should be correct and coherent.
The quiz is worth 5 points.

Does the sequence whose n^{th} term is $a_n = n\sin(\frac{1}{n})$ converge? If so, what is the limit of the sequence? Explain your answer very thoroughly.

Answer: We compute $\lim\limits_{n\to\infty} a_n$. The first factor goes to infinity and the second factor goes to zero. This is an indeterminate form (that is, one can not tell the answer from the form of the problem alone – one must dig more deeply into to the functions that comprise our problem). We will turn the problem into a quotient and use L'hopital's rule. We see that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}}.$$

The top and the bottom both go to zero, so we may apply L'hopital's rule to see that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\frac{-1}{n^2}\cos(\frac{1}{n})}{\frac{-1}{n^2}} = \lim_{n\to\infty} \frac{\cos(\frac{1}{n})}{1} = 1.$$

We conclude that

$$\boxed{\text{the sequence } \{a_n\} \text{ converges to } 1.}$$