

PRINT Your Name: _____

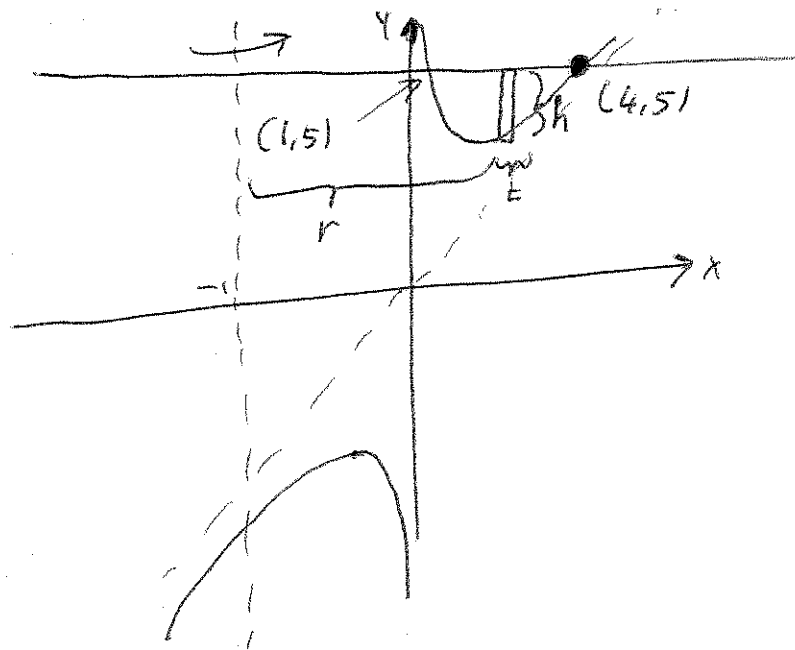
Quiz 6 — February 23, 2011 — Section 4 — 9:05-9:55 recitation.

Remove everything from your desk except this page and a pencil or pen.

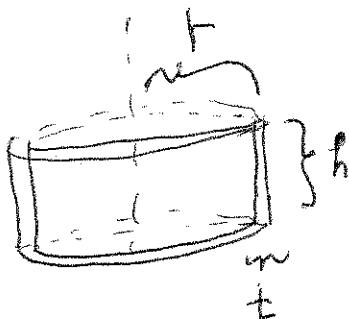
Circle your answer. Show your work. The quiz is worth 5 points.

Consider the region bounded by $y = 5$ and $y = x + \frac{4}{x}$. Revolve this region about the line $x = -1$. Find the volume of the resulting solid.

Find the intersection points by solving $5 = x + \frac{4}{x}$. Multiply by x to obtain $5x = x^2 + 4$. This is $0 = x^2 - 5x + 4$ or $0 = (x - 4)(x - 1)$. The intersection points are $(1, 5)$ and $(4, 5)$. (Notice that both points are on both graphs.) The graph of $y = x + \frac{4}{x}$ is positive when x is positive and negative when x is negative. When x is positive and near zero, the graph goes to infinity. When x is big the graph is close to $y = x$.



Partition the x -axis from $x = 1$ to $x = 4$. Draw the rectangle with x -coordinate x . Rotate the rectangle to get a shell



of volume $2\pi rht$, where $t = dx$, r is the distance from the rectangle to the axis of revolution, so $r = x - (-1) = x + 1$; and h is the height of the rectangle. We see that h is the y -coordinate at the top minus the y -coordinate at the bottom; that is, $h = 5 - (x + \frac{4}{x})$. The shell has volume

$$2\pi rht = 2\pi(x+1) \left(5 - \left(x + \frac{4}{x} \right) \right) dx = 2\pi \left(4x - x^2 + 1 - \frac{4}{x} \right) dx.$$

The volume of the solid is

$$\begin{aligned} 2\pi \int_1^4 \left(4x - x^2 + 1 - \frac{4}{x} \right) dx &= 2\pi \left(2x^2 - \frac{x^3}{3} + x - 4 \ln x \right) \Big|_1^4 \\ &= \boxed{2\pi \left(32 - \frac{64}{3} + 4 - 4 \ln 4 - \left(2 - \frac{1}{3} + 1 \right) \right)} \end{aligned}$$