

PRINT Your Name: _____

Quiz 7 — September 8, 2015

Remove everything from your desk except this page and a pencil or pen.

The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work.** Your work must be correct and coherent. **Check your answer.**

Find $\int \frac{dx}{\sqrt{x^2+16}} dx$.

Answer: Let $x = 4 \tan \theta$. We compute $x^2 + 16 = 16 \tan^2 \theta + 16 = 16(\tan^2 \theta + 1) = 16 \sec^2 \theta$. We also compute $dx = 4 \sec^2 \theta d\theta$. So,

$$\begin{aligned} \int \frac{dx}{\sqrt{x^2+16}} &= \int \frac{4 \sec^2 \theta d\theta}{4 \sec \theta} = \int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C = \ln \left| \frac{\sqrt{x^2+16}}{4} + \frac{x}{4} \right| + C \\ &= \ln |\sqrt{x^2 + 16} + x| - \ln 4 + C = \boxed{\ln |\sqrt{x^2 + 16} + x| + K}, \end{aligned}$$

where $K = -\ln 4 + C$.

Check. The derivative of the proposed answer is

$$\frac{\frac{2x}{2\sqrt{x^2+16}} + 1}{\sqrt{x^2 + 16} + x} = \frac{x + \sqrt{x^2 + 16}}{\sqrt{x^2 + 16}(\sqrt{x^2 + 16} + x)} = \frac{1}{\sqrt{x^2 + 16}} \checkmark$$