

PRINT Your Name: \_\_\_\_\_

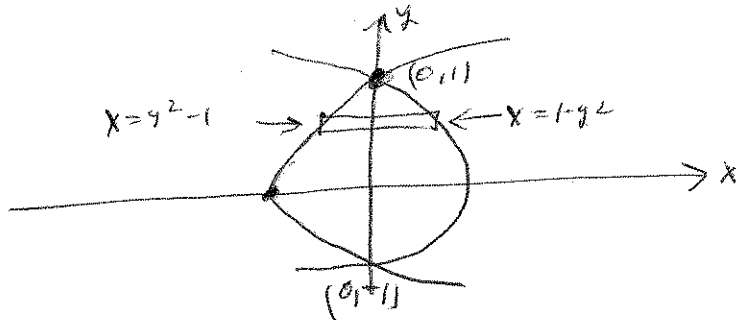
Quiz 6 — September 23, 2011 — Section 7 — 10:10 — 11:00

Remove everything from your desk except a pencil or pen.

The quiz is worth 5 points.

Find the area between  $x = 1 - y^2$  and  $x = y^2 - 1$ . You must draw a meaningful picture.

Answer:



The intersection occurs when  $1 - y^2 = y^2 - 1$ ; so  $0 = 2y^2 - 2$  or  $0 = 2(y^2 - 1)$  or  $0 = 2(y - 1)(y + 1)$ . Thus,  $y = -1$  or  $y = 1$ . Of course, if  $y = 1$ , then  $x = 0$  and if  $y = -1$ , then  $x = 0$ . Chop the  $y$ -axis from  $y$  equals  $-1$  to  $x = 1$ . Over each piece of the  $y$ -axis, we draw a rectangle of area  $h \cdot t$ , where  $h$  is the height and  $t$  is the thickness. We have  $t = \Delta y$ . We must express  $h$  in terms of  $y$ . We see that  $h$  is equal to the  $x$ -value at the right of the rectangle minus the  $x$ -value at the left of the rectangle. Thus,  $h = (1 - y^2) - (y^2 - 1) = -2(y^2 - 1)$ . The area of the rectangle is  $h \cdot t = 2(y^2 - 1)\Delta y$ . The area of the region is

$$\begin{aligned} -2 \int_{-1}^1 (y^2 - 1) dy &= -2 \left( \frac{y^3}{3} - y \right) \Big|_{-1}^1 = -4 \left( -\frac{2}{3} \right) \\ &= \boxed{\frac{8}{3}}. \end{aligned}$$