PRINT Your Name:_____

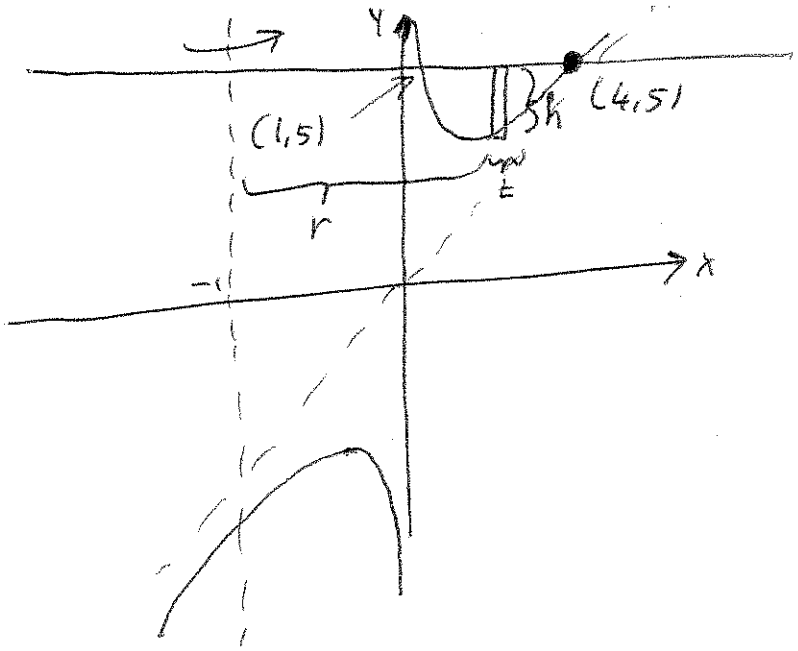
Quiz 6 — February 23, 2011 – Section 3 – 8:00-8:50 recitation.

Remove everything from your desk except this page and a pencil or pen.

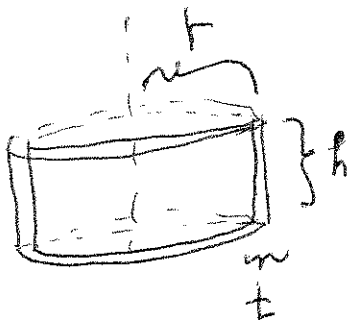
Circle your answer. **Show your work.** The quiz is worth 5 points.

Consider the region bounded by $y = 5$ and $y = x + \frac{4}{x}$. Revolve this region about the line $x = -1$. Find the volume of the resulting solid.

Find the intersection points by solving $5 = x + \frac{4}{x}$. Multiply by x to obtain $5x = x^2 + 4$. This is $0 = x^2 - 5x + 4$ or $0 = (x-4)(x-1)$. The intersection points are $(1, 5)$ and $(4, 5)$. (Notice that both points are on both graphs.) The graph of $y = x + \frac{4}{x}$ is positive when x is positive and negative when x is negative. When x is positive and near zero, the graph goes to infinity. When x is big the graph is close to $y = x$.



Partition the x-axis from $x = 1$ to $x = 4$. Draw the rectangle with x-coordinate x. Rotate the rectangle to get a shell



of volume $2\pi rht$, where $t = dx$, r is the distance from the rectangle to the axis of revolution, so $r = x - (-1) = x + 1$; and h is the height of the rectangle. We see that h is the y-coordinate at the top minus the y-coordinate at the bottom; that is, $h = 5 - (x + \frac{4}{x})$. The shell has volume

$$2\pi rht = 2\pi(x+1)\left(5 - \left(x + \frac{4}{x}\right)\right) dx = 2\pi\left(4x - x^2 + 1 - \frac{4}{x}\right) dx.$$

The volume of the solid is

$$2\pi \int_1^4 \left(4x - x^2 + 1 - \frac{4}{x}\right) dx = 2\pi\left(2x^2 - \frac{x^3}{3} + x - 4\ln x\right)\Big|_1^4$$

$$= \boxed{2\pi\left(32 - \frac{64}{3} + 4 - 4\ln 4 - \left(2 - \frac{1}{3} + 1\right)\right)}$$