

PRINT Your Name: _____

Quiz 6 — September 3, 2015

Remove everything from your desk except this page and a pencil or pen.

The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work.** Your work must be correct and coherent. **Check your answer.**

Find $\int \tan^5 x dx$.

Answer: Use the identity $\tan^2 x + 1 = \sec^2 x$.

$$\begin{aligned}\int \tan^5 x dx &= \int \tan^3 x (\sec^2 x - 1) dx = \int (\tan^3 x \sec^2 x - \tan^3 x) dx \\ &= \int (\tan^3 x \sec^2 x - \tan x (\sec^2 x - 1)) dx \\ &= \int (\tan^3 x \sec^2 x - \tan x \sec^2 x) dx + \int \tan x dx.\end{aligned}$$

In the first integral, let $u = \tan x$; so $du = \sec^2 x dx$. In the second integral, let $w = \cos x$; so $dw = -\sin x dx$. It follows that

$$\begin{aligned}\int \tan^5 x dx &= \int (u^3 - u) du + \int \frac{dw}{w} = u^4/4 - u^2/2 - \ln |w| + C \\ &= \boxed{\frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} - \ln |\cos x| + C}.\end{aligned}$$

Check. The derivative of the proposed answer is

$$\tan^3 x \sec^2 x - \tan x \sec^2 x - \frac{-\sin x}{\cos x} = \tan^3 x \sec^2 x - \tan x (\sec^2 x - 1) =$$

$$\tan^3 x \sec^2 x - \tan^3 x = \tan^3 x (\sec^2 x - 1) = \tan^5 x. \checkmark$$