

PRINT Your Name: \_\_\_\_\_

**Quiz 5 — September 22, 2010 — Section 9 — 10:10 — 11:00**

**Remove everything from your desk except this page and a pencil or pen.**

**Circle** your answer. **Show your work.** **Check your answer.**

The quiz is worth 5 points.

Find  $\int \frac{xe^x}{\sqrt{1+e^x}} dx$ .

**Answer:** Use integration by parts. Let  $u = x$  and  $dv = \frac{e^x}{\sqrt{1+e^x}} dx$ . It follows that  $du = dx$  and  $v = 2\sqrt{1+e^x}$ . The integral is equal to

$$2x\sqrt{1+e^x} - 2 \int \sqrt{1+e^x} dx.$$

Let  $w = \sqrt{1+e^x}$ . It follows that  $dw = \frac{e^x dx}{2\sqrt{1+e^x}}$ . In other words,  $\frac{2w dw}{w^2-1} = dx$ . Our integral is equal to

$$\begin{aligned} &= 2x\sqrt{1+e^x} - 2 \int \frac{2w^2}{w^2-1} dw = 2x\sqrt{1+e^x} - 4 \int \frac{w^2}{w^2-1} dw \\ &= 2x\sqrt{1+e^x} - 4 \int \frac{(w^2-1)+1}{w^2-1} dw = 2x\sqrt{1+e^x} - 4 \int 1 + \frac{1}{w^2-1} dw. \end{aligned}$$

Use partial fractions to see that

$$\frac{1}{w^2-1} = \frac{1}{2} \left[ \frac{1}{w-1} - \frac{1}{w+1} \right].$$

Our integral is

$$\begin{aligned} &= 2x\sqrt{1+e^x} - 4 \int 1 + \frac{1/2}{w-1} - \frac{1/2}{w+1} dw \\ &= 2x\sqrt{1+e^x} - 4 \left( w + \frac{1}{2} \ln|w-1| - \frac{1}{2} \ln|w+1| \right) + C \\ &= 2x\sqrt{1+e^x} - 4 \left( \sqrt{1+e^x} + \frac{1}{2} \ln|\sqrt{1+e^x}-1| - \frac{1}{2} \ln|\sqrt{1+e^x}+1| \right) + C \\ &= \boxed{2(x-2)\sqrt{1+e^x} - 2 \ln|\sqrt{1+e^x}-1| + 2 \ln|\sqrt{1+e^x}+1| + C.} \end{aligned}$$

Check. The derivative of the proposed answer is

$$\frac{2(x-2)e^x}{2\sqrt{1+e^x}} + 2\sqrt{1+e^x} - 2 \frac{\frac{e^x}{2\sqrt{1+e^x}}}{\sqrt{1+e^x}-1} + 2 \frac{\frac{e^x}{2\sqrt{1+e^x}}}{\sqrt{1+e^x}+1}$$

$$\begin{aligned}
&= \frac{(x-2)e^x}{\sqrt{1+e^x}} + \frac{2(1+e^x)}{\sqrt{1+e^x}} + \frac{e^x}{\sqrt{1+e^x}} \left[ \frac{-1}{\sqrt{1+e^x}-1} + \frac{1}{\sqrt{1+e^x}+1} \right] \\
&= \frac{(x-2)e^x}{\sqrt{1+e^x}} + \frac{2(1+e^x)}{\sqrt{1+e^x}} + \frac{e^x}{\sqrt{1+e^x}} \left[ \frac{-(\sqrt{1+e^x}+1) + (\sqrt{1+e^x}-1)}{1+e^x-1} \right] \\
&= \frac{(x-2)e^x}{\sqrt{1+e^x}} + \frac{2(1+e^x)}{\sqrt{1+e^x}} + \frac{e^x}{\sqrt{1+e^x}} \left[ \frac{-2}{e^x} \right] \\
&= \frac{(x-2)e^x}{\sqrt{1+e^x}} + \frac{2(1+e^x)}{\sqrt{1+e^x}} + \frac{-2}{\sqrt{1+e^x}} = \frac{xe^x}{\sqrt{1+e^x}}. \checkmark
\end{aligned}$$