

PRINT Your Name: \_\_\_\_\_

**Quiz 5 — September 16, 2011 — Section 8 — 11:15 — 12:05**

**Remove everything from your desk except a pencil or pen.**

Circle your answer. **Show your work.** Your work should be correct and coherent. **CHECK** your answer.

The quiz is worth 5 points.

Find  $\int \frac{dx}{1+e^x} dx$ .

**Answer:** Let  $u = e^x$ . It follows that  $du = e^x dx$ ; hence  $\frac{du}{u} = dx$ . The original integral is equal to  $\int \frac{du}{u(1+u)}$ . We use the technique of partial fractions to write

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{1+u}.$$

So  $1 = A(1+u) + Bu$ . Plug in  $u = 0$  to learn  $A = 1$ . Plug in  $u = -1$  to learn  $B = -1$ . We conclude  $\frac{1}{u(u+1)} = \frac{1}{u} - \frac{1}{1+u}$ . (This is correct.) The original integral is equal to

$$\begin{aligned} \int \left( \frac{1}{u} - \frac{1}{1+u} \right) du &= \ln|u| - \ln|1+u| + C = \ln|e^x| - \ln|1+e^x| + C \\ &= \ln e^x - \ln(1+e^x) + C = \boxed{x - \ln(1+e^x) + C}. \end{aligned}$$

(We used the fact that  $e^x$  and  $1+e^x$  are both always positive.)

**Check:** The derivative of the proposed answer is:

$$1 - \frac{e^x}{1+e^x} = \frac{1}{1+e^x}. \checkmark$$