PRINT Your Name:_____

Quiz 5 — February 16, 2011 – Section 4 – 9:05-9:55 recitation.

Remove everything from your desk except this page and a pencil or pen.

Circle your answer. **Show your work.** **Check** your answer.
The quiz is worth 5 points.

Find $\int \sec x \tan^2 x \, dx$. Check your answer.

We do this integral by parts. Let $u = \tan x$ and $dv = \sec x \tan x dx$. We compute $du = \sec^2 x dx$ and $v = \sec x$. We have

$$\int \sec x \tan^2 x \, dx = \sec x \tan x - \int \sec^3 x dx = \sec x \tan x - \int \sec x(\tan^2 x + 1)dx$$

$$= \sec x \tan x - \int \sec x \tan^2 x dx - \int \sec x dx.$$

Add $\int \sec x \tan^2 x dx$ to both sides

$$2\int \sec x \tan^2 x \, dx = \sec x \tan x - \int \sec x dx = \sec x \tan x - \ln|\sec x + \tan x| + C.$$

Divide by 2 to see that

$$\boxed{\int \sec x \tan^2 x \, dx = \frac{1}{2}(\sec x \tan x - \ln|\sec x + \tan x|) + C.}$$

Check. The derivative of the proposed answer is

$$\frac{1}{2}\left(\sec x \sec^2 x + \sec x \tan x \tan x - \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}\right)$$

$$= \frac{1}{2}\left(\sec x \sec^2 x + \sec x \tan x \tan x - \sec x\right)$$

$$= \frac{1}{2}\sec x\left(\sec^2 x + \tan^2 x - 1\right) = \frac{1}{2}\sec x\left(\tan^2 x + \tan^2 x\right). \checkmark$$