PRINT Your Name:____________________

Quiz 5 — February 14, 2014 – Section 7 – 12:00 – 12:50

Remove everything from your desk except this page and a pencil or pen.
The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work. Your work must be correct and coherent. Check your answer.**
The quiz is worth 5 points.

Find $\int \frac{1}{x\sqrt{4x^2+1}}dx$.

Answer: Let $2x = \tan\theta$. It follows that $4x^2+1=\sec^2\theta$ and $2dx=\sec^2\theta d\theta$. The original problem is equal to

$$\int \frac{\frac{1}{2}\sec^2\theta}{\frac{1}{2}\tan\theta\sec\theta}d\theta = \int \frac{\sec\theta}{\tan\theta}d\theta = \int \frac{1}{\sin\theta}d\theta = \int \csc\theta d\theta = \int \frac{\csc\theta(\csc\theta+\cot\theta)}{\csc\theta+\cot\theta}d\theta$$

$$= -\ln|\csc\theta+\cot\theta| + C = -\ln|\tfrac{\sqrt{4x^2+1}}{2x}+\tfrac{1}{2x}| + C = -\ln|\tfrac{\sqrt{4x^2+1}+1}{2x}| + C$$

$$= \boxed{-\ln(\sqrt{4x^2+1}+1)+\ln|2x|+C.}$$

(I have drawn a right triangle, one of whose angles is called θ. The side opposite θ has length $2x$, the adjacent has length 1, and the hypotenuse has length $\sqrt{4x^2+1}$. So $\tan\theta = \frac{\text{Op}}{\text{Adj}} = 2x$ and I can easily read of the rest of the trig functions evaluated at θ in terms of x.)

Check. The derivative of

$$-\ln(\sqrt{4x^2+1}+1)+\ln(2x)$$

is

$$-\frac{\frac{8x}{2\sqrt{4x^2+1}}}{\sqrt{4x^2+1}+1} + \frac{2}{2x} = -\frac{4x}{(\sqrt{4x^2+1}+1)\sqrt{4x^2+1}} + \frac{1}{x}$$

$$= \frac{-4x^2+(\sqrt{4x^2+1}+1)\sqrt{4x^2+1}}{x(\sqrt{4x^2+1}+1)\sqrt{4x^2+1}} = \frac{-4x^2+(4x^2+1+\sqrt{4x^2+1})}{x(\sqrt{4x^2+1}+1)\sqrt{4x^2+1}}$$

$$= \frac{1+\sqrt{4x^2+1}}{x(\sqrt{4x^2+1}+1)\sqrt{4x^2+1}} = \frac{1}{x\sqrt{4x^2+1}}. \checkmark$$