PRINT Your Name:____________________

Quiz 4 — February 9, 2011 – Section 4 – 9:05-9:55 recitation.

Remove everything from your desk except this page and a pencil or pen.

Circle your answer. **Show your work.** **Check** your answer.
The quiz is worth 5 points.

Find $\int \frac{1}{x\sqrt{4x^2+1}}\,dx$**. Check your answer.**

Let $2x = \tan\theta$. It follows that $2dx = \sec^2\theta d\theta$ and $4x^2+1 = \tan^2\theta + 1 = \sec^2\theta$. The integral is

$$\left(\frac{1}{2}\right)\int \frac{\sec^2\theta d\theta}{\frac{\tan\theta}{2}\sec\theta} = \int \frac{\sec^2\theta d\theta}{\tan\theta\sec\theta} = \int \frac{\sec\theta d\theta}{\tan\theta} = \int \frac{\sec\theta\tan\theta d\theta}{\tan^2\theta}$$

$$= \int \frac{\sec\theta\tan\theta d\theta}{\sec^2\theta - 1}.$$

Let $u = \sec\theta$. It follows that $du = \sec\theta\tan\theta d\theta$. The integral is

$$= \int \frac{du}{u^2-1} = \frac{1}{2}\int\left(\frac{1}{u-1} - \frac{1}{u+1}\right)du = \frac{1}{2}\left(\ln|u-1| - \ln|u+1|\right)$$

$$= \frac{1}{2}\left(\ln|\sec\theta - 1| - \ln|\sec\theta+1|\right) = \boxed{\frac{1}{2}\left(\ln|\sqrt{4x^2+1}-1| - \ln|\sqrt{4x^2+1}+1|\right)}.$$

Check. The derivative of the proposed answer is

$$\frac{1}{2}\left(\frac{\frac{8x}{2\sqrt{4x^2+1}}}{\sqrt{4x^2+1}-1} - \frac{\frac{8x}{2\sqrt{4x^2+1}}}{\sqrt{4x^2+1}+1}\right)$$

$$= \frac{2x}{\sqrt{4x^2+1}}\left(\frac{1}{\sqrt{4x^2+1}-1} - \frac{1}{\sqrt{4x^2+1}+1}\right)$$

$$= \frac{2x}{\sqrt{4x^2+1}}\left(\frac{\sqrt{4x^2+1}+1}{(\sqrt{4x^2+1}-1)(\sqrt{4x^2+1}+1)} - \frac{\sqrt{4x^2+1}-1}{(\sqrt{4x^2+1}+1)(\sqrt{4x^2+1}-1)}\right)$$

$$= \frac{2x}{\sqrt{4x^2+1}}\left(\frac{2}{(4x^2+1)-1}\right) = \frac{2x}{\sqrt{4x^2+1}}\frac{2}{4x^2} = \frac{1}{\sqrt{4x^2+1}}\frac{1}{x}. \checkmark$$